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A NEW GOODNESS-OF-FIT TEST FOR THE GAMMA DISTRIBUTION BASED ON SAMPLE SPACINGS FROM COMPLETE AND CENSORED SAMPLES

THESIS

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First Lieutenant

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A NEW GOODNESS-OF-FIT TEST FOR THE GAMMA DISTRIBUTION WITH KNOWN SHAPE PARAMETER BASED ON SAMPLE SPACINGS FROM COMPLETE AND CENSORED SAMPLES

THESIS

Presented to the Faculty of the Graduate School of Engineering
of the Air Force Institute of Technology
Air Education and Training Command
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research

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Preface

This thesis aims to improve the power and effectiveness of goodness-of-fit tests for the gamma distribution. The critical value tables were generated for a new test statistic, Z^*. A power study was conducted to determine the power of the new test and compare it with the power of K-S, Cv-M, and A-D test statistics.

I could not have undertaken such an effort without the guidance of an experienced professor in the field, my thesis advisor Dr. Albert H. Moore. His remarkable insight directed me throughout every phase of this research, and his guidance saved me precious time that I desperately needed. Thank you Dr. Moore for sharing your wisdom and patiently teaching me the finer points of goodness-of-fit procedures.

My appreciations go to Lt. Col. Paul F. Auclair for his careful editing and helpful comments. He was most helpful at revising this thesis for preparing it for publication. I also express my appreciation to Dr. Joseph P. Cain for his assistance and comments.

I offer special thanks to the instructors and personnel of Department of Operational Sciences. Their understanding of the hardships of working toward a graduate degree in a foreign language helped us catch up with the other students.

I would like to thank my fellow classmates, especially Ozzy and Güneş, for their support during those hard times. I am also grateful to my dear parents for the sacrifice they made for years, to raise a child that will manage to get out of his village and do something that they will be proud of.

Finally, I would like to dedicate this thesis effort to my sweetheart Paula. She supported me along such a challenge even though we were thousands of miles apart.

Hüseyin DUMAN

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Abstract

A new goodness-of-fit test based on spacings was applied to the gamma distribution with known shape parameter. The size of samples varied between 5 and 35. The critical value tables were generated for the Z^* test statistic for complete and censored samples. The critical values were obtained for five different significance levels: 0.20, 0.15, 0.10, 0.05, and 0.01. An extensive power study, containing $50,000$ Monte Carlo runs, was conducted using nine alternative distributions, H_a.

It was observed that the Z^* test statistic was more powerful against certain alternatives which are less skewed than the gamma distribution with a given shape parameter.

A regression between the critical values and the sample size, shape parameter, significance levels and degree of censoring was established. Regression functions for each shape parameter studied in this thesis were also presented.

The power of the Z^* test statistic is compared to the powers of the competing test statistics (K-S, W^2, and A-D). Tables of power comparison are presented for two different shape parameters.

This thesis reveals that the Z^* test statistic is a directional test. This feature may be utilized to attain higher power values by coupling the Z^* and the A-D test statistics in a sequential test.

A NEW GOODNESS-OF-FIT TEST FOR THE GAMMA DISTRIBUTION WITH KNOWN SHAPE PARAMETER BASED ON SAMPLE SPACINGS FROM COMPLETE AND CENSORED SAMPLES

I. INTRODUCTION

The Air Force uses statistical models to describe the failure patterns of various mechanical and electronic components in its weapon systems. These models are used especially in reliability and life span analyses. The same type of analysis is done in many other large organizations in the aerospace industry. These studies assume that the sampled data follow a particular statistical distribution function and base their computations on this assumption. The quality of any model's predictions will only be as good as the quality of the assumptions on which the model is based.

Goodness-of-fit tests provide statistical procedures to assess the validity of the distributional assumptions. These tests check the likelihood that an observed sample could have been generated from a population defined by a proposed distribution. In this study, a goodness-of-fit test statistic that hasn't been applied to the gamma distribution was studied. This test statistic is referred to as Z^*, and is based on order statistics.

1.1 Background

The gamma family of distributions is widely used in reliability studies and life span analyses. Like the Weibull, lognormal and inverse Gaussian distributions, the gamma distribution is a positively skewed distribution that is frequently used in

reliability applications and life span analyses (4:113-121). In particular, the gamma distribution is often used to model the time-to-failure distribution of electrical, mechanical, and combined systems (5:243).

An essential and often forgotten aspect of the modelling exercise, the goodness-of-fit test assesses the validity of model and data correspondence. Since inferences about the greater population are based on the sample data set, the distribution modelling the time-to-failure must adequately represent the data. The goodness of fit test is designed to reject the initial assumption of the population distribution if the test statistic exceeds some critical value. The 'power' of a goodness-of-fit test is the probability of rejecting the initial hypothesis when the underlying population model is incorrect (17:431-434).

The goodness-of-fit tests and parameter estimation methods developed for the gamma distribution generally rely on maximum likelihood estimation, minimum distance estimation or a combination of the two. Some powerful goodness-of-fit tests were developed by Viviano (33) and by Özmen (24) for the gamma distribution. Viviano used maximum likelihood estimation, while Özmen used both maximum likelihood and minimum distance estimation in his procedure. Coppa (6) used the Z^* test statistic, a goodness-of-fit test based on the spacings between adjacent order statistics, in developing an effective test for the Weibull distribution. A goodness-of-fit test based on spacings for the gamma distribution that can be applied to complete and censored samples is not currently available.

1.2 An Overview of the Goodness-of-Fit Tests

Statistical tests can determine if the sample data correspond to a hypothesized failure model. By observation, time to failure sample data can be collected and then compared to a theoretical probability distribution. The tests that determine if the hypothesized distribution fits the sample data are simply called goodness-of-fit

tests. Several of the classical goodness-of-fit tests that are most commonly used in statistical studies include the:

- Chi-square (χ^2), goodness-of-fit test
- Kolmogorov-Smirnov ($K - S$) goodness-of-fit test
- Anderson-Darling (A^2) goodness-of-fit test
- Cramer von Mises (W^2) goodness-of-fit test

If these tests indicate an adequate fit, the hypothesized distribution can be used to predict the failure rates of the sampled object or weapon system. Such a study enhances planning for the wartime effectiveness of that particular weapon system and improves the accuracy of the logistics plan.

The objective of a statistical test is to evaluate a hypothesis concerning the values of one or more of the population parameters. We generally have a hypothesized distribution that we wish to support or disprove. If we can find sufficient evidence from the sample data to refute our theory, we conclude that the null hypothesis is false. The converse of the null hypothesis is called the alternative hypothesis.

We decide between the null hypothesis, H_0, and the alternative hypothesis, H_a, by evaluating a test statistic based on sample data. If the value of our test statistic falls in a certain range or rejection region, we conclude that H_0 is false and that the sample more than likely did not come from the population predicted by the null hypothesis. All statistical tests of hypothesis work in the same way and are composed of the same basic elements:

- Null hypothesis, H_0
- Alternative hypothesis, H_a
- Test Statistic
- Rejection region

The functioning parts of the statistical test are the test statistic and the rejection region. The rejection region specifies the value of the test statistic for which the null hypothesis H_0 is rejected.

There are two types of errors that can be made when reaching a decision about the null hypothesis:

- A type I error is made if H_0 is rejected when H_0 is true. The probability of a Type I error is denoted by α.
- A type II error is made if H_0 is accepted when H_a is true. The probability of a Type II error is denoted by β.

Thus, α and β measure the risks associated with making an erroneous decision; As such, they provide a practical measure of the efficiency of a test of hypothesis. The power of the test, denoted by $(1-\beta)$, is the probability of rejecting the null hypothesis, H_0, when it is false. In simple terms, we can explain the power of a test as the probability that the test statistic will reject the null hypothesis, H_0, when it is false; that is, when the sample is not from the hypothesized population. In this research, we observed whether or not the test statistic accepted samples from Weibull, beta and lognormal distributions as samples from a gamma distribution. In the ideal case, a test statistic is expected to reject the samples from alternative distributions, and to accept the samples that are from the true distribution.

1.3 The Gamma Distribution

There are several failure models and associated goodness-of-fit tests that attempt to satisfy the requirement of modelling component failure patterns. This thesis considered only the three parameter gamma distribution.

The probability density function (pdf) of the three parameter gamma distribution is;

$$f(x) = \frac{(x-\delta)^{\alpha-1} \ e^{\frac{-(x-\delta)}{\beta}}}{\beta^{\alpha} \ \Gamma(\alpha)} \qquad (for \quad \alpha > 0, \beta > 0; x > \delta) \qquad (1.1)$$

The gamma distribution depends on three parameters α, β and γ. α is the shape parameter, β is the scale parameter, and δ is the location parameter. The shape parameter determines the general appearance, or shape, of the distribution. The location parameter determines the guaranteed life, and the scale parameter determines the relative scale of the distribution. In most cases, even if the location parameter is not $\delta = 0$, the data can be transformed to the case where $\delta = 0$. More detailed information about the gamma distribution will is included in the Literature Review.

1.4 Problem Statement

There are several goodness-of-fit tests developed for the three parameter gamma distribution with unknown location and scale parameter based on maximum likelihood estimation, minimum distance estimation, or a combination of both estimation techniques. The objective of this research was to define a new goodness-of-fit test based on spacings by obtaining critical values for the Z^* test statistic and to compare the power of the Z^* test with the previous test used by Viviano (33).

1.5 Focus of Research

Tiku and Singh (29) derived a new test statistic (Z*) by modifying Mann's (S) statistic. This new test statistic is based on the ratio of the differences of adjacent order statistics and is computationally simpler than previously developed statistics based on maximum likelihood or minimum distance estimation (32). Evaluation of the Z^* test statistic indicates that it has a higher power for skewed distributions.

When random variables X_i are arranged in ascending order of magnitude, $X_{i;n}$ is said to be the i^{th} order statistic in a sample of size n. If a test to estimate the failure times is continued until all sample specimens have failed, the sample is complete and consists of the ordered observations of failure times: $X_1 \ldots, X_n$. If the test is terminated with p specimens still operating, where $p < n$, this sample is censored and consists of $(n - p)$ observations of failure times, $X_1 \ldots, X_{n-p}$.

This research initially focused on complete samples. When the Z^* test proved to be more powerful than its predecessors, censored samples were included within the scope of this research.

1.6 Assumptions

This research assumed that the shape parameter of the three parameter gamma distribution is known. The shape parameter might be estimated from the complete or the censored sample. The parameters assumed to be unknown are the location and scale parameters.

1.7 Scope

The main objective of this research was to obtain critical values for a new test statistic, and to check its power through an extensive power study. The power study compared the performance of the Z^* test with the alternative goodness-of-fit tests, (K-S, Cv-M and A-D), used by Viviano (33). The power study compared the performance of the new test statistic in only complete samples. Since there was no previous research on goodness-of-fit tests for fitting censored samples to a gamma distribution, the power study for censored samples was not compared to the power values of any other test statistic.

The objectives of this thesis were to:

- Generate rejection tables at various alpha levels for the Z^* test statistic from complete and censored samples.
- Conduct a power comparison between the Z^* and the previous test statistics for complete samples.
- Present the power values of the Z^* test statistic for censored samples.
- Investigate a regression function that estimates the critical values of the test based on sample size, shape parameter and level of significance, α.

II. LITERATURE REVIEW

2.1 The Three Parameter Gamma Distribution

A random variable X has a gamma distribution if its probability density function is

$$f(x) = \frac{(x-\delta)^{\alpha-1}\ e^{\frac{-(x-\delta)}{\beta}}}{\beta^{\alpha}\ \Gamma(\alpha)} \qquad (for \quad \alpha > 0, \beta > 0; x > \delta) \tag{2.1}$$

where α is the shape parameter, β is the scale parameter, and δ is the location parameter. The gamma distribution is a positively skewed distribution for small values of the shape parameter and it is close to symmetric for large values of the shape parameter (3:1-9). Figure 2.1 depicts the broad range of shapes the gamma distribution can assume by varying the shape parameter.

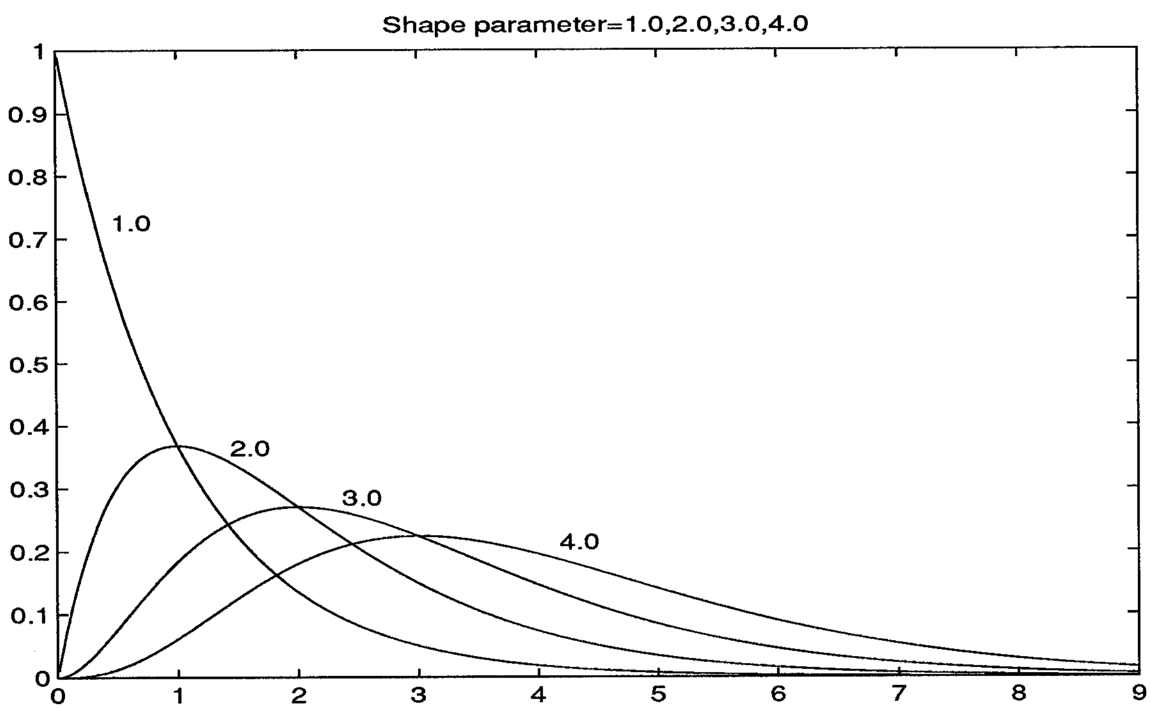

Figure 2.1 The Gamma Distributions with shape parameters, $\alpha = 1.0, 2.0, 3.0, 4.0$.

If X is a random variable following a gamma distribution with parameters α and β, then mean and the variance of X are respectively

$$\mu = E(X) = \alpha\,\beta \tag{2.2}$$

and

$$\sigma^2 = V(X) = \alpha\beta^2. \tag{2.3}$$

The standard form of the distribution is obtained by setting $\beta = 1$ and $\delta = 0$, yielding

$$f(x) = \frac{x^{\alpha-1}\ e^{-x}}{\Gamma(\alpha)} \qquad (x \geq 0) \tag{2.4}$$

If $\alpha = 1$, this is an *exponential distribution.* If α is a positive integer, it is an *Erlang distribution.*

The probability integral of the gamma distribution in standard form is

$$P[X \leq x] = [\Gamma(\alpha)]^{-1} \int_0^x t^{\alpha-1}\ e^{-t} dt. \tag{2.5}$$

and is referred to as the *incomplete gamma function ratio.* The quantity

$$\Gamma_x(\alpha) = \int_0^x t^{\alpha-1}\ e^{-t} dt \tag{2.6}$$

is sometimes called an *incomplete gamma function.* Since the incomplete gamma function ratio depends on x and α, and it would be natural to use a notation representing it as a function of these variables.

However, Pearson found it more convenient to substitute $x\alpha^{-\frac{1}{2}}$ in place of x for tabulation purposes (15). He defined the incomplete gamma function as

$$I(u, \alpha - 1) = \frac{1}{\Gamma(\alpha)} \int_0^{u\sqrt{\alpha}} t^{\alpha-1} e^{-t} dt. \tag{2.7}$$

The primary significance of the (standard) gamma distribution in statistical theory lies in the fact that if $U_1, U_2, \ldots, U_\nu$ are independent unit normal variables, the pdf of $\sum_{j=1}^{\nu} U_j^2$ is of form given in Equation 2. 1, with $\alpha = \frac{1}{2}\nu; \beta = 2; \delta = 0$. This particular form of gamma distribution is called a *chi-square distribution with* ν *degrees of freedom.* The corresponding random variable is often denoted by χ_ν^2. It is clear that $\frac{1}{2}\sum_{j=1}^{\nu} U_j^2$ has a standard gamma distribution with $\alpha = \frac{1}{2}\nu$.

Expressed symbolically:

$$f_{\chi_\nu^2}(x^2) = \{2^{\frac{1}{2}\nu}\,\Gamma(\frac{1}{2}\nu)\}^{-1}(x^2)^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x^2} \qquad (x^2 \geq 0) \tag{2.8}$$

Although in the above definition, ν must be an integer, the distribution stated above is also called a "χ^2 *distribution with* ν *degrees of freedom*" if ν is any positive number (15:111).

If X is a chi-square random variable with ν degrees of freedom, the mean and variance of X are given by

$$\mu = E(X) = \nu \tag{2.9}$$

$$\sigma^2 = V(X) = 2\,\nu. \tag{2.10}$$

Plots of χ^2 distributions with different degrees of freedom are presented in Figure 2. 2.

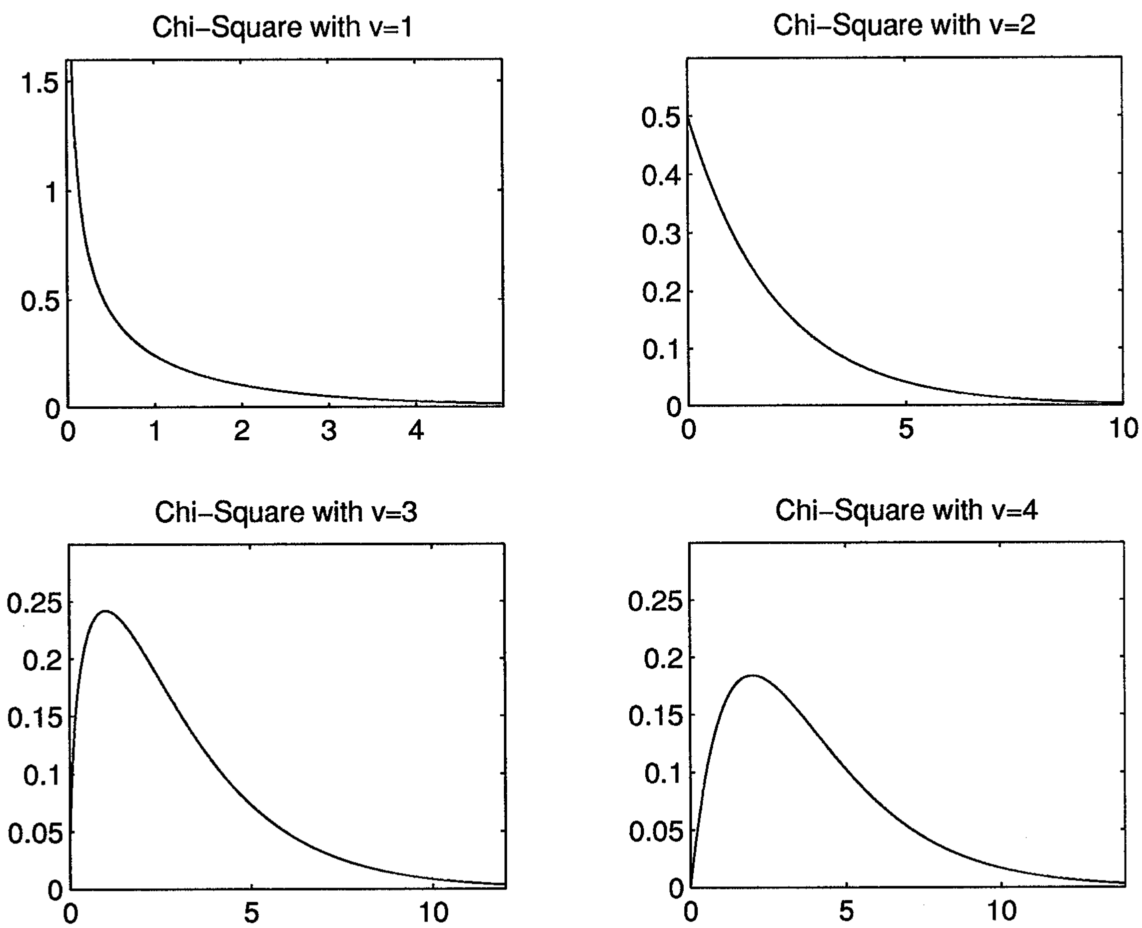

Figure 2.2 Graphs of χ^2 Distributions with $\nu = 2\alpha$.

In the case that the shape parameter α is not an integer there is no closed form of gamma distribution. When α is an integer, we obtain the following

$$F(x) = \begin{cases} 1 - e^{-\frac{x}{\beta}} \sum_{j=0}^{\alpha-1} \frac{(\frac{x}{\beta})^j}{j!} & \text{if} \quad x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

with shape parameter $\alpha > 0$, and scale parameter $\beta > 0$.

2.2 The Applications of the Gamma Distribution

There are many applications of the gamma distribution in lifetime data analysis. The length of time between malfunctions for aircraft engines has a skewed frequency distribution, as do the lengths of time between arrivals at a supermarket checkout queue. The populations associated with such random variables are often adequately modeled by the gamma distribution (21:164-168). Because the gamma distribution is mathematically tractable and can assume a wide range of shapes, it is widely used in reliability studies for life-testing and in statistical survival analysis for describing the distribution of elapsed time to some generic event of interest.

For distributions representing times to failure, the hazard function is often known as the failure-rate function. Quite usually this failure-rate function follows a fairly standard pattern with respect to time for most technological devices.

A typical pattern of this sort is illustrated in Figure 2.3. Here the failure-rate function is plotted against time and can be seen to fall into three distinct phases.

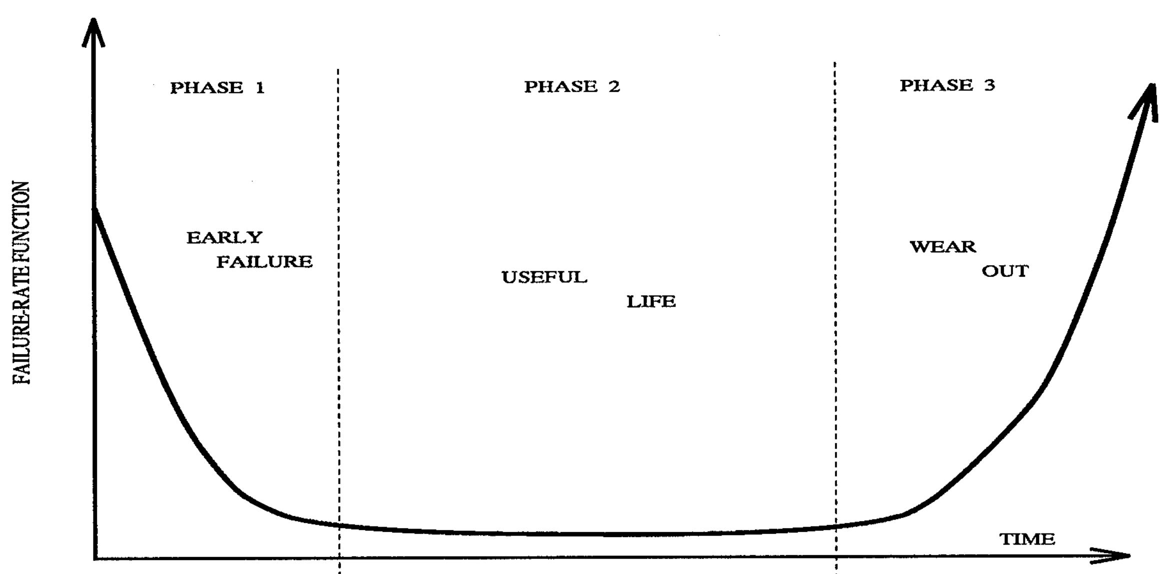

Figure 2.3 Typical failure-rate characteristic for engineering devices.

This plôt is called the 'bathtub curve'. The first phase represents a pattern of failure which typically arises from initial production, test or assembly faults. The last phase illustrates the effects of aging when the device is beginning to wear out and the rate of failures tends to increase. In between the first and last phases is a phase which may be termed the 'useful life' where the failure-rate function remains either sensibly constant or follows a relatively slow change in value.

These three phases can be modeled with three different groups of statistical distributions. The first phase can be represented by a gamma hazard function with a shape parameter close to 0.5. The second phase can be represented by an exponential hazard function, which is a special case of the gamma hazard function with the shape parameter $\alpha = 1.0$. The last phase can be modeled by normal distribution. Manufacturers attempt to prolong Phase 2, since this phase has lowest failure rates. For simplicity, most studies assume that the devices under study have survived the first phase and are operating in their second, or 'useful life,' phase are broken-in already which puts the device into the second phase. The exponential distribution approximates the steady failure-rate of Phase 2 fairly well. For this reason, the exponential distribution is used routinely to model the times to failure, while the time period between failures is modeled by gamma distribution (8:535-541).

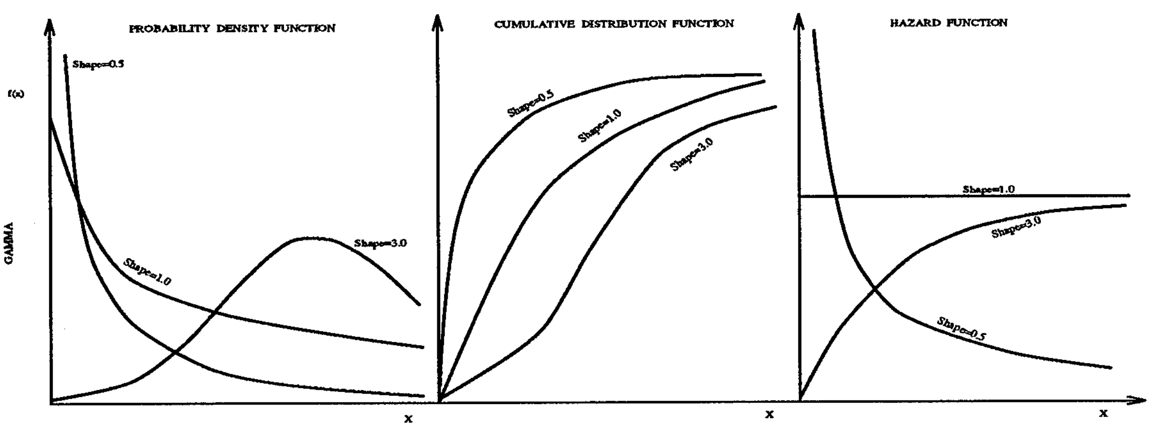

Figure 2.4 PDF, CDF and Hazard Functions of the Gamma Distribution.

As seen in Figure 2.4, the hazard function of the gamma distribution with shape parameter $\alpha = 0.5$ may represent Phase I, while the hazard function of the gamma distribution with shape parameter $\alpha = 1.0$ (i.e. exponential distribution) may represent Phase 2 of the failure-rate function of a device.

Similarly, we can consider a human being to be a biological unit whose failure is called a 'death'. In this case, the failure-rate function is called 'the force of mortality'. Figure 2.5 shows the similarity between the probability of death for human subjects and the failure rate function for electronic components. It shows that the studies in reliability analysis can also be applied in human life-span analysis (16).

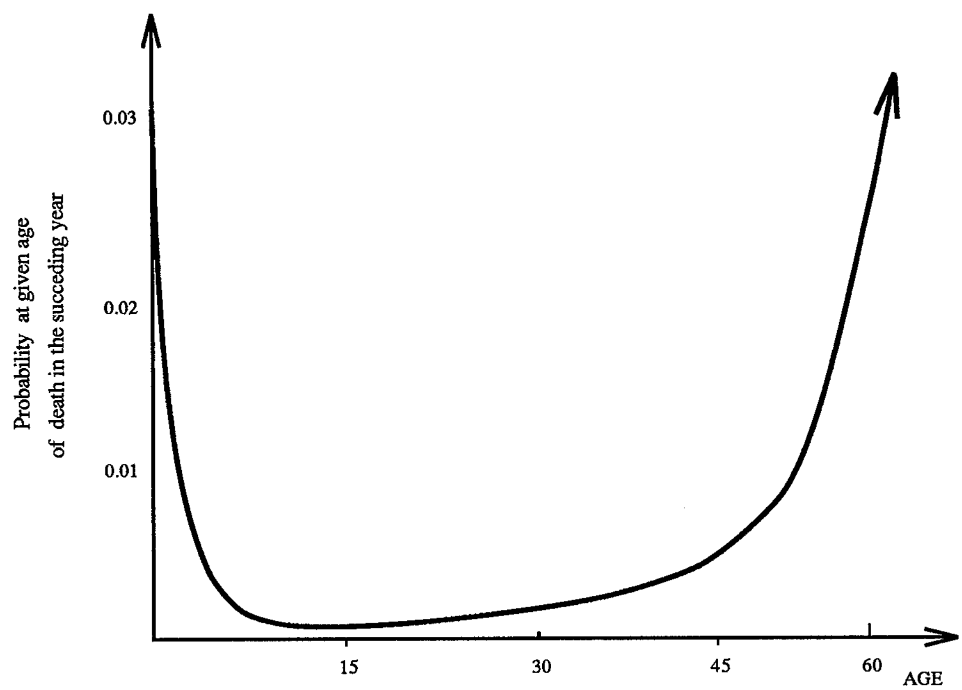

Figure 2.5 Death-rate characteristic for males living in England.

Although the gamma distribution plays a major role in lifetime and reliability studies, it is clearly not restricted to such applications. In one novel study, Matis et al. used the gamma distribution extensively as a transit-time model in a stochastic

analysis to predict animal abundance at multiple locations and to predict the density of an Africanized honey bee population (13). A similar approach can be used in predicting the spread of a virus, a biological gas or a disease. The transit-time required for the agent to cover a certain area of interest can be modelled by gamma distribution.

2.3 Classical Goodness-of-Fit Tests

In general, formal goodness-of-fit tests seek to determine whether a sample data set can be hypothesized to have come from a commonly known probability distribution. Since inferences about the general population are based on the smaller data set, it is important to test the fit of the data to the population(17:121).

2.3.1 Chi-squared Type Tests.

In 1900, Karl Pearson abandoned the assumption that biological populations were normally distributed, and introduced his system of distributions to provide other models. The need to test the fit of his proposed models resulted in the Chi-squared test statistic, which is among today's most used statistical procedures. Pearson's idea was to reduce the general problem of testing fit to a multinomial setting by basing a test on a comparison of observed cell counts versus their expected values under the hypothesis H_0 to be tested. This reduction of data to grouped sets discards some information, making tests of this type somewhat less powerful than other classes of tests of fit (17:179-182).

χ^2 test statistic is given by

$$\chi^2 = \sum_{j-1}^{k} \frac{(N_j - np_j)^2}{np_j} \tag{2.11}$$

χ^2 value is smaller than the critical value if the hypothesized distribution adequately represents the sample. But in comparison with other test statistics χ^2 test has lower power especially at small sample sizes.

2.3.2 **Empirical Distribution Function Tests.** Goodness of fit tests based on the empirical distribution function (EDF) are discussed extensively by Stephens (27). A great number of statistics have been proposed for testing the null hypothesis H_0 based on the idea of measuring the distance between the theoretical cdf $F_\theta(x)$ and the empirical cdf $G_n(x)$. Goodness-of-fit tests based on empirical distribution function include the Kolmogorov-Smirnov test (K-S), Cramer-von Mises test (Cv-M) and Anderson-Darling (A-D) tests (28:44).

2.3.3 **Kolmogorov-Smirnov Test Statistic.** The K-S test statistic is (27);

$$\begin{aligned} D^+ &= \max\left[(i/n) - Z_i\right] \qquad & 1 \le i \le n \\ D^- &= \max\left[Z_i - (i-1)/n\right] \qquad & 1 \le i \le n \\ D &= \max\left[D^+,\ D^-\right] \end{aligned}$$

where $Z_i = F_\theta(x_i)$.

2.3.4 **Cramer-von Mises Test Statistic.** The Cv-M test statistic is denoted by W^2,

$$W_\psi^2(G_n, F_\theta) = \int_{-\infty}^{+\infty} \left[G_n(x) - F_\theta(x)\right]^2 \psi\left[F_\theta(x)\right] dF_\theta(x) \tag{2.12}$$

with special interest given to the case where $\psi(\cdot) \equiv 1$.

Stephens' computational formula is (27);

$$W^2 = \sum_{i=1}^{n} \left[\frac{F_\theta(x_i)_i - (2i-1)}{2n}\right]^2 + \frac{1}{12n} \tag{2.13}$$

2.3.5 **Anderson-Darling Test Statistic.** The Anderson-Darling statistic is one of the more powerful empirical distribution function based tests of fit in a wide range of circumstances. It is a special case of the Cramer-von Mises distance, where $\psi(u) = 1/[u(1-u)],\ 0 < u < 1$ and takes the form

$$A_n^2(G_n, F_\theta) = \int_{-\infty}^{+\infty} [G_n(x) - F\theta(x)]^2\ [F_\theta(x)\,[1 - F_\theta(x)]]^{-1}\ dF_\theta(x) \qquad (2.14)$$

Stephens' computational formula is (27);

$$A^2 = -\left[\left[\sum_{i=1}^{n} (2i-1)\,[\ln Z_i + \ln(1 - Z_{n+1-i})]\right]/n\right] - n \qquad (2.15)$$

where $Z_i = F_\theta(x_i)$.

Tests based on these statistics are distribution free; they do not depend on the hypothesized distribution if all the parameters are completely specified.

2.4 Tests Based on Order Statistics

The subject of order statistics deals with properties and applications of ordered random variables and of functions of these variables. When random variables X_i are arranged in ascending order of magnitude, then $X_{(i;n)}$ is said to be the i^{th} order statistic in a sample of size n. To illustrate the application of order statistics to goodness of fit tests, consider a life test on a certain electrical component. A random selection of n specimens is made from the population of items available. This sample is placed on test. If the test is continued until all sample specimens have failed, the sample is said to be complete, and it consists of the ordered observations of failure times: $X_{(1;n)}$ to $X_{(n;n)}$. In the common case where testing is terminated with survivors, the sample is said to be censored.

Some frequently encountered functions of order statistics are:

- The extremes $X_{(1;n)}$ and $X_{(n;n)}$
- The range $W = X_{(n;n)} - X_{(1;n)}$
- The extreme deviate from the sample mean, defined as Max($X_{(i;n)} - \bar{X}$)

These statistics play an important role in practical applications. The extremes arise in the study of floods and droughts, as well as in breaking strength and fatigue failure studies. The range is widely used by quality control practitioners to provide quick estimates of the standard deviation in a normal distribution. The extreme deviate from sample mean is also used in fatigue failure analysis (5).

Some goodness-of-fit tests using order statistics were developed for several other distributions. Cressie (7) summarizes the H, I, and D test statistics in his study of the uniform distribution. If we define the sequence of i^{th}-order gaps for an ordered sample of n observations on an uniform random variable as

$$G_n^{(m)} = U_{(n+m)} - U_{(n)} \qquad (n = 0, 1, \ldots, n+1-m), \tag{2.16}$$

where U_i is the i^{th} uniform order statistic, $U_{(0)} = 0$, and $U_{(n+1)} = 1$. Then H statistic can be represented as

$$H_n^{(m)} = \sum_{n=0}^{n+1-m} h(nG_n^{(m)}) \qquad (m \leq n+1), \tag{2.17}$$

for those $h(\cdot)$ satisfying certain regularity conditions.

This test statistic was modified also by Pino (25) as D statistic:

$$D_n^{(m)} = \sum_{i=0}^{k_n} k(nG_i^{(m)}) \qquad (m \leq n+1), \tag{2.18}$$

where $k_n = [(n+1)/m] - 1$. Finally, Holst came up with the L test statistic

$$L_n^{(m)} = \sum_{n=0}^{n+1-m} \log(nG_n^{(m)}) \qquad (m \leq n+1), \tag{2.19}$$

These tests were developed for the uniform distribution. Some of these test statistics can be applied to nonuniform distributions, too (7).

Wells et al. (34) studied some tests of fit using spacings statistics with estimated parameters. They assumed that sample data were collected from an unknown underlying continuous cumulative distribution.They present some theorems and proofs in their study showing that those test statistics have the same asymptotic distribution in the case when parameters must be estimated from the sample as in the case when parameters are specified (34).

Some powerful distributional tests based on sample spacings were studied by Hall (9). He revised the test statistics that were studied by Pino (25), and Cressie (7). Hall especially focuses on the S statistic also developed by Pino (25), which is a modification of D statistic, represented as

$$S_n \equiv \sum_{k=1}^{[n/m]} h\left[(n/m)(X_{n,km} - X_{n,(k-1)m})\right] \tag{2.20}$$

Another test statistic based on spacings was developed by Mann (18). She used S statistic for the two parameter Weibull distribution, and showed that it was as powerful as the test statistics based on the empirical distribution function. S statistic is the ratio of a linear combination of the sum of order statistics divided by the expected values of a normalized sum of order statistics from a sample.

The advantage of the S statistic was that it was relatively easy to calculate and the parameters of the Weibull distribution did not need to be known or estimated other than the shape parameter. S statistic is represented as

$$S = \frac{\sum_{i=\frac{n}{2}+1}^{n-1} G_i}{\sum_{i=1}^{n-1} G_i} \tag{2.21}$$

where G_i is the ratio defined by

$$G_i = \frac{X_{i+1} - X_i}{\mu_{(i+1)} - \mu_{(i)}} \tag{2.22}$$

In this notation X_i represents the i^{th} ordered observation from a sample of size n. Mann et al. used this new test statistic for the two parameter and three parameter Weibull distributions for known shape parameter. When they compared the power of the S statistic, it was observed that the power of the S statistic was at least as good as its competitors (22).

Mann et al. also introduced a derivative of S statistic as M test statistic. This test statistic was used for extreme value distribution (19). M test statistic is given by

$$M = \frac{\left(\frac{n}{2}\right) \sum_{i=\frac{n}{2}+1}^{n-1} G_i}{\left(\frac{n-1}{2}\right) \sum_{i=1}^{n/2} G_i} \tag{2.23}$$

where

$$G_i = \frac{X_{i+1} - X_i}{\mu_{(i+1)} - \mu_{(i)}} \tag{2.24}$$

A modification of Mann's S statistic was developed by Tiku (29) as Z^* test statistic. Tiku modified Mann's S statistic by changing the coefficients of the terms in the linear combination. This statistic is the ratio of the sums of ordered observations divided by sums of their expected values, with different limits on the summations

when compared to Mann's S statistic (32). It was shown as

$$Z^* = \frac{2\sum_{i=1}^{n-2}(n-1-i)\,G_i}{(n-2)\sum_{i=1}^{n-1}\,G_i} \tag{2.25}$$

where G_i is as defined in Equation 221. The numerator of G_i is the difference between ordered observations, X_{i+1} and X_i. The denominator of G_i is the difference between the expected values of the $(i+1)^{th}$ and i^{th} order statistics, denoted by $\mu_{(i+1:n)} - \mu_{(i:n)}$.

Tiku applied the Z^* test statistic to the normal distribution. He showed that the power of the Z^* test statistic was generally more powerful against asymmetric alternatives, and less powerful against symmetric alternatives when compared with tests of normality like Shapiro-Wilk and Cramer-von Mises test statistics.

Balakrishnan (2) contributed to Z^* test statistic by calculating the power of this test statistic for several distributions. He showed that the approximate power value obtained asymptotically and the simulated power value obtained by 4,000 Monte Carlo runs were very close. He presented the power results for several statistical distributions for complete samples. He found the Z^* test statistic to be more powerful than Durbin D^* statistic, and Shapiro-Wilk W^* statistic when all distributions are considered. More detailed information on Z^*-statistic is presented in the Methodology chapter.

2.5 The Concept and Types of Censoring

Censoring creates special problems in the analysis of lifetime data. Broadly speaking, censoring occurs when exact lifetimes are known for only a portion of the individuals under study.

When an individual has his lifetime censored at L, we will call L the censoring time for that observation. Formally, an observation is said to be right censored at L if the exact value of the observation is not known but only that it is greater than or equal to L. Similarly, an observation is said to be left censored at L if it is known only

that the observation is less than or equal to L. Right censoring is very common in lifetime data, but left censoring is fairly rare. For convenience, the term 'censoring' will be used meaning in all instances 'right censoring'. There are three predominant types of censoring; Type I, Type II, and Random censoring (17).

2.5.1 **Type I Censoring.** Sometimes experiments are run over a fixed time period in such a way that an individual's lifetime will be known exactly only if it is less than some predetermined value. In such situations the data are said to be Type I or time censored. For example, in a life test experiment n items may be placed on test, but a decision made to terminate the test after a time L has elapsed. Lifetimes will then be known exactly only for those items that fail by time L. Type I censoring frequently arises in medical research where, for example, a decision is made to terminate a study at a date on which not all the individuals' lifetimes will be known. Let's suppose an AIDS research is made on the lifetime of n AIDS patients who have the disease, and after L years the research is stopped. During this period r of the patients will be dead, for $0 \leq r \leq n$, and $n - r$ of them will be living. We will determine the exact lifetimes of the patients who died in the L year period, but we don't have an idea about the lifetimes of the survivors (17).

2.5.2 **Type II Censoring.** In contrast, a sample of size n is said to be Type II censored when only its r smallest lifetimes are observed $(1 \leq r \leq n)$. Experiments involving Type II censoring are often used in lifetime testing. As an example, a total of n items is placed on test, but instead of continuing until all n items have failed, the test is terminated at the time the r^{th} item fails. Such tests can save time and money, since it could take a very long time for all the items to fail in some instances.

The statistical treatment of Type II censored data is, at least in principle, straightforward. The number of observations r is usually decided before the data are collected. The resulting data consist of the r smallest lifetimes $T_{(1)} \leq T_{(2)} \leq$

$\cdots \leq T_{(r)}$ out of a random sample of n *iid* lifetimes $T_1 \cdots T_n$ presumed to have a continuous distribution with pdf $f(t)$ and survivor function $S(t)$. It follows from general results on order statistics that the joint pdf of $T_{(1)}, \ldots, T_{(r)}$ is

$$f_n(t_{(1)}, t_{(2)}, \ldots, t_{(n)}) = \frac{n!}{(n-r)!} f(t_{(1)}) \cdots f(t_{(r)}) [S(t_{(r)})]^{n-r} \tag{2.26}$$

For any given parametric model statistical inference can be based on the likelihood function given in Equation 2.24.

To summarize, the difference between Type II and Type I censoring is the termination of the experiment. Type II censoring stops the test when r of the n individuals fail, while Type I stops the test after a predetermined time limit (17).

2.5.2.1 **Progressive Type II Censoring.** A generalization of Type II censoring is progressive Type II censoring. In this case, the first r_1 failures in a sample of n items are observed: then n_1 of the remaining $(n - r_1)$ unfailed items are removed from the experiment, leaving $(n - r_1 - n_1)$ items still present. When further r_2 items have failed, n_2 of the still unfailed are removed, and so on. The experiment terminates after some prearranged series of repetitions of this procedure. In progressive censoring some of the entities are taken out of the sample for some reason. In a sample of AIDS study, if an entity dies of cancer, this entity is not taken into consideration as an observation since the cause of death is not what we would want to observe. This type of censoring is not widely used in lifetime analysis, because it makes the calculations highly complicated, even though it is widely used for the lifespan analysis of the patients in hospitals. Considering that our objective is generating critical values for gamma distribution with a censoring technique that is widely used and easy to calculate, the progressive Type II censoring was not included in this research(17).

2.5.3 **Random Censoring.** Censoring times are often effectively random. For example, in a medical trial patients may enter the study in a more or less random fashion, according to their time of diagnosis. If the study is terminated at some prearranged date, then censoring times, that is the length of time from an individual's entry into the study until the termination of the study, are random. For inference purposes one often works conditionally on the observed censoring times, proceeding as though the censoring were Type I, but the process by which the data were generated needs to be considered in order to justify this. The assumption of a random censoring mechanism is also a useful device for investigating the properties of certain procedures(17).

2.6 Summary

The need to model failure time data motivated the discussion of the gamma distribution. The wide variety of shapes assumed by both its density and hazard functions makes the gamma distribution suitable for broad range of applications. However, the appropriateness of the gamma distribution to model a particular set of data depends on how well it conforms to the data.

We typically assess how well a distribution represents the data through goodness-of-fit testing. While advances have been made in goodness-of-fit tests, many of these advances have not been applied to the gamma distribution. This research applied a goodness-of-fit test based on spacings to the gamma distribution with known shape parameter.

Chapter 3 explains the methodology employed in this research, while Chapter 4 presents the results and analysis of the research. Finally, Chapter 5 closes this thesis with a set of conclusions and recommendations.

III. METHODOLOGY

The goal of this thesis was to develop a new goodness-of-fit test for the gamma distribution where the shape parameter α is known. The scale parameter β and the location parameter γ do not need to be known or estimated.

3.1 The Z* Test Statistic

The test statistic used throughout this thesis was a derivative of the one used by Tiku (32). It is denoted by Z^* and is given by

$$Z^* = \frac{2\sum_{i=1}^{n-2}(n-1-i)\,G_i}{(n-2)\sum_{i=1}^{n-1}\,G_i} \tag{3.1}$$

where G_i is the ratio defined by

$$G_i = \frac{X_{i+1} - X_i}{\mu_{(i+1:n)} - \mu_{(i:n)}} \tag{3.2}$$

The numerator of G_i, $X_{i+1} - X_i$, is the difference between the i^{th} and the $(i+1)^{st}$ ordered observations of the sample. This difference is referred to as the i^{th} gap of the sample. There are $n-1$ gaps in a sample of size n. The values of the order statistics, X_i, are taken directly from the sample.

The denominator of G_i, $\mu_{(i+1:n)} - \mu_{(i:n)}$, is the difference between expected value of i^{th} and the $(i+1)^{st}$ order statistics from the standard gamma distribution with scale parameter β, shape parameter α, and location parameter δ. There are $(n-1)$ μ_{dif} values in a sample of size n (31).

In simplified form, the ratio G_i becomes

$$G_i = \frac{X_{i+1} - X_i}{\mu_{dif_i}} \tag{3.3}$$

The expected value of the m^{th} order statistic from a sample of n observations from a gamma distribution with shape parameter α and scale parameter θ is given by

$$E(x_{m,n}; \theta, \alpha) = \frac{\theta n}{\Gamma(\alpha)} \binom{n-1}{m-1} \int_0^\infty \left[\frac{\Gamma(\alpha; z)}{\Gamma(\alpha)}\right]^{m-1} \left[1 - \frac{\Gamma(\alpha; z)}{\Gamma(\alpha)}\right]^{n-m} z^\alpha \, e^{-z} \, dz, \tag{3.4}$$

where $\Gamma(\alpha; z)$ is the incomplete Gamma function defined by (10)

$$\Gamma(\alpha; z) = \int_0^z t^{\alpha-1} \, e^{-t} \, dt \tag{3.5}$$

3.2 Computation of Critical Values for Z*

Monte Carlo Simulation was used to obtain the critical values of the Z^* test statistic. The specific steps involved in the procedure were:

1. For a fixed sample size n and shape parameter α, random deviates from the gamma distribution were generated using the IMSL subroutine RNGAM. All gamma deviates were generated with scale parameter equal to one and location parameter equal to zero.

2. The n random gamma deviates were additively scaled by ten.

3. The n scaled gamma deviates were sorted in ascending order using IMSL subroutine SVRGN. The values obtained after the sort operation were the X_i values used in the numerator of G_i.

4. The differences between the expected values of the order statistics, μ_{dif} were obtained separately and read into the FORTRAN program for use in the denominator of G_i.

5. The G_i values were input into the summation formula to compute the Z^* test statistic value for this one sample of size n.

6. Steps 1 to 5 were repeated 10,000 times, thus generating 10,000 independent Z^* statistics.

7. The 10,000 Z^* statistics were sorted in ascending order using the IMSL subroutine SVRGN.

8. The 80th, 85th, 90th, 95th and the 99th percentile were found by linear interpolation. These percentiles comprise the critical values for the test statistic.
9. These computations were made for 10 different seed values, and their mean values were taken for each percentile.

Figure 3.1 summarizes the process used to generate the critical values for the Z^* test statistic. This process was repeated for different sample sizes and shape parameters. For shape parameters $\alpha = 0.5,\ 1.0,\ \ldots, 4.0$, the sample size ranged from 5 to 35 in multiples of 5. In addition, a sample size of 40 augmented the $\alpha = 0.5$ case to illustrate the effect of a higher sample size.

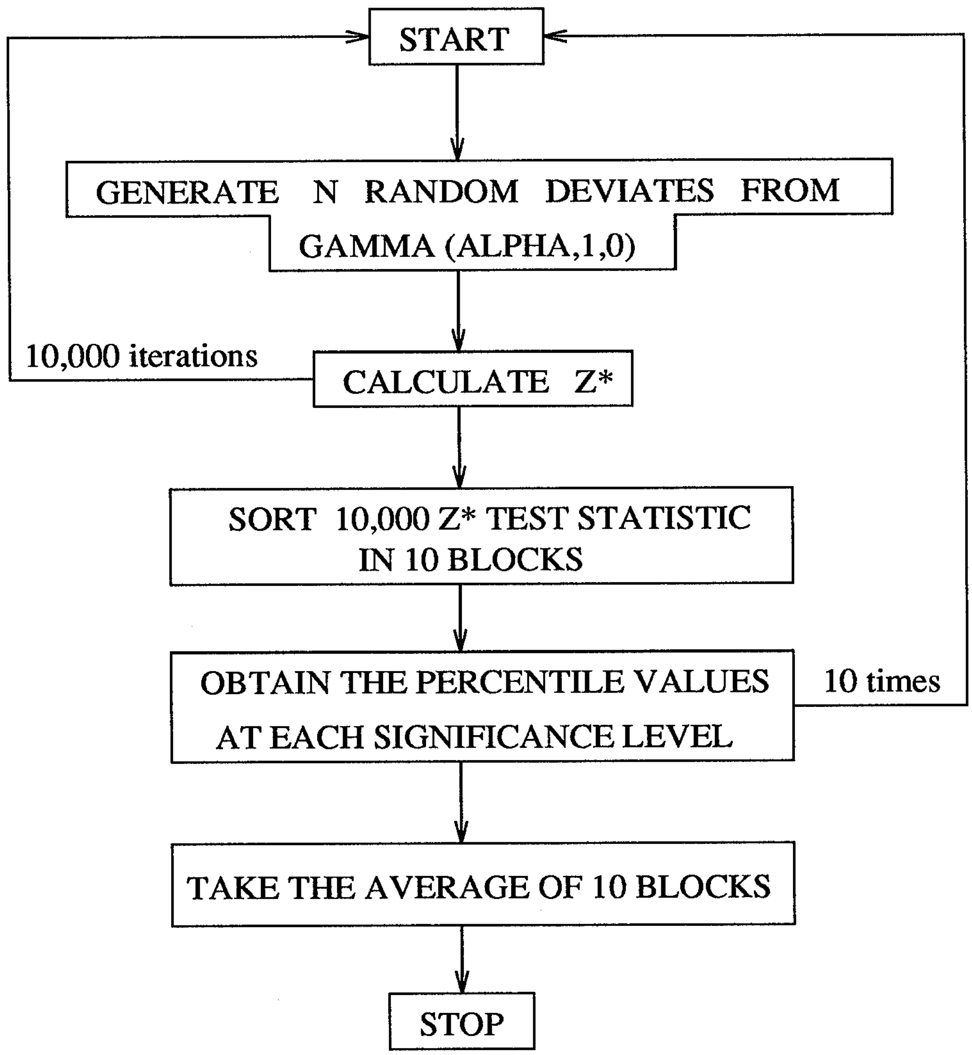

Figure 3.1 Generation of Critical Values for Z^*

3.3 Power Study of the Z^* Statistic

The performance of any goodness-of-fit test statistic is known as the *Power of the Test.* It is defined as the probability of rejecting the null hypothesis, H_0, when an alternate hypothesis H_a is true. In this thesis, nine alternate distributions were considered in the power study for the Z^* test statistic. A direct comparison with the prominent competitors was made for shape parameter=1.0 and sample size 5, 15 and 25. Power figures for this case were obtained from Viviano (33). Power data for all other combinations of shape parameter α and sample size n were accomplished independently via the Monte Carlo method.

3.3.1 The Distributions $\mathbf{H_0}$ and $\mathbf{H_a}$.

The null hypothesis for the power study was that sample deviates follow a gamma distribution with shape parameter α. The values of α used were $\alpha = 0.5,\ 1.0,\ 1.5,\ 2.0,\ 3.0$, and 4.0. The 9 alternative hypotheses, H_a, identified by Distribution numbers 2 thru 10 in Table 3. 1.

Table 3.1 Statistical Distribution Functions Used for the Power Study

Distribution	Distribution Name and Parameters
1	Gamma with shape parameter=original
2	Gamma with shape parameter=1.5
3	Gamma with shape parameter=2.5
4	Gamma with shape parameter=4.0
5	Weibull with shape parameter=2.0
6	Weibull with shape parameter=3.0
7	Lognormal with w=0.0, p=2.0
8	Lognormal with w=1.0, p=1.0
9	Beta with p=2.0, q=2.0
10	Beta with p=2.0, q=3.0

For comparison, some of the distributions used in the power study are plotted against gamma distribution. Figure 3.2 indicates the comparison between Weibull distribution and the gamma distributions.

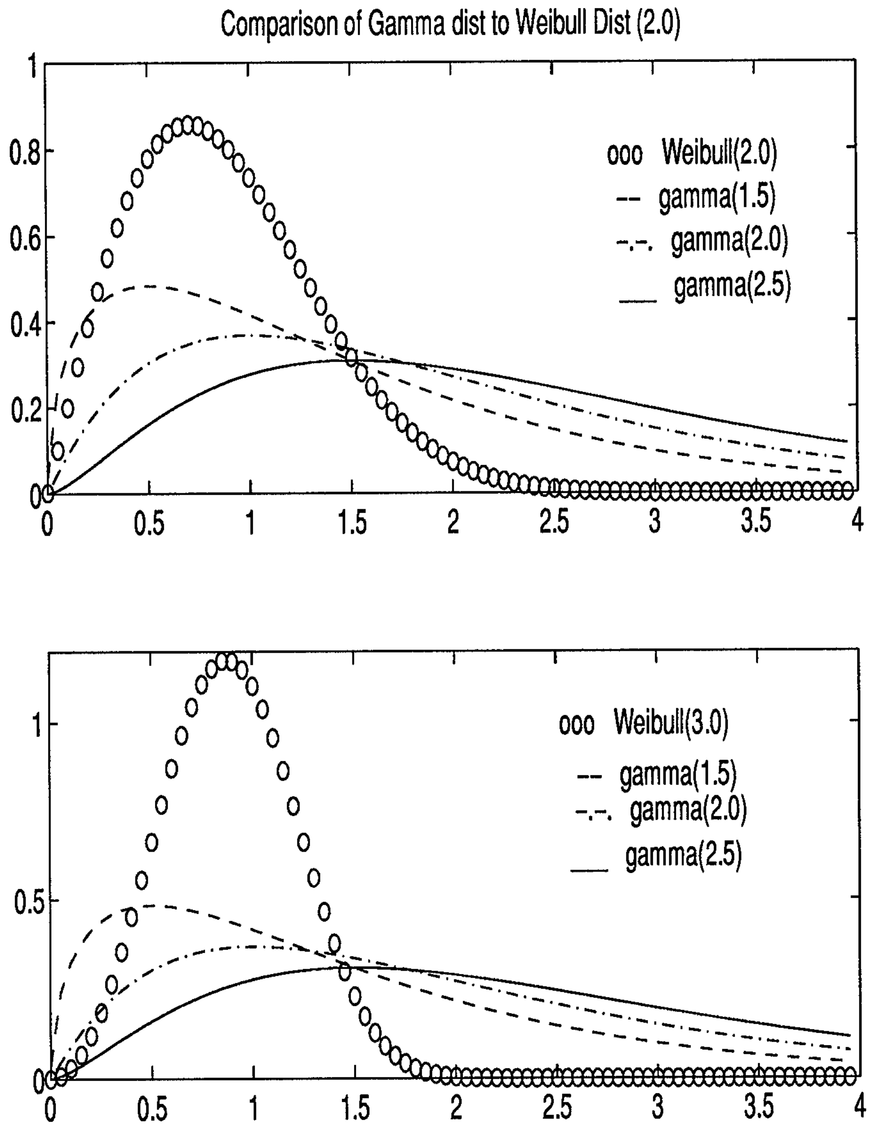

Figure 3.2 Gamma vs Weibull Distributions in the Power Study.

Figure 3.3 compares the lognormal distribution to the gamma distributions with different shape parameters.

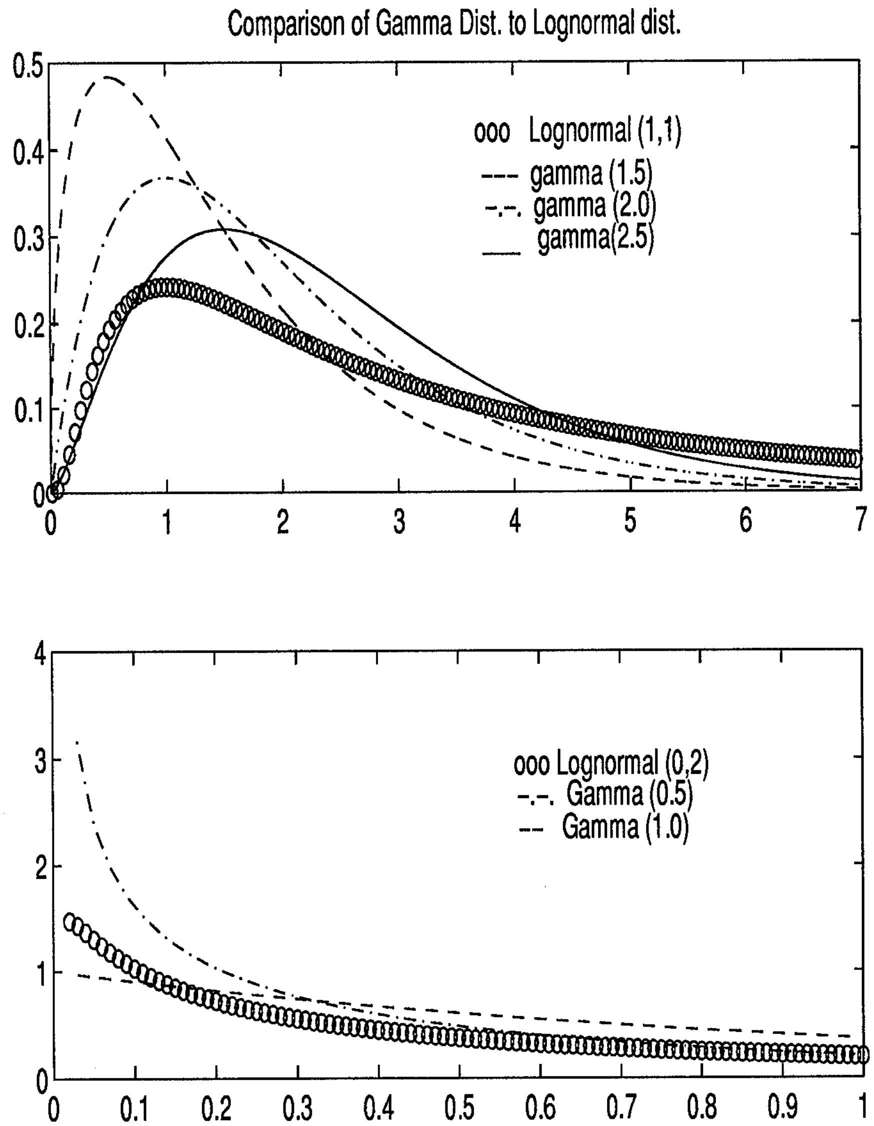

Figure 3.3 Gamma vs Lognormal Distributions in the Power Study.

3.3.2 **Power Study Process.** The power study process was very similar to the critical value determination procedure. A Monte Carlo simulation was accomplished with 5000 iterations and a different random number seed than what was used in the critical value program. The steps in the process were:

1. Random deviates from H_a for a fixed sample size n were generated using the appropriate IMSL subroutine.

2. Deviates were additively scaled by 10 and sorted in ascending order. The values obtained at this point are the X_i values used in the numerator of G_i.

3. The differences between the expected values of the order statistics for a sample of size n, shape α and scale parameter equal to one were read into the program to establish the denominator of G_i.

4. The Z^* test statistic was calculated for each iteration.

5. The Z^* value attained was compared to the critical value at each of five α levels.

6. The number of times the Z^* statistic exceeds the critical value was counted for each of the five α levels. Exceeding the critical value is equivalent to rejecting H_0 at the corresponding significance level.

The process was repeated for each different alternate distribution, H_a, at various sample sizes.

Figure 3.4 presents the flow diagram followed during the generation of tables of power against alternative distribution.

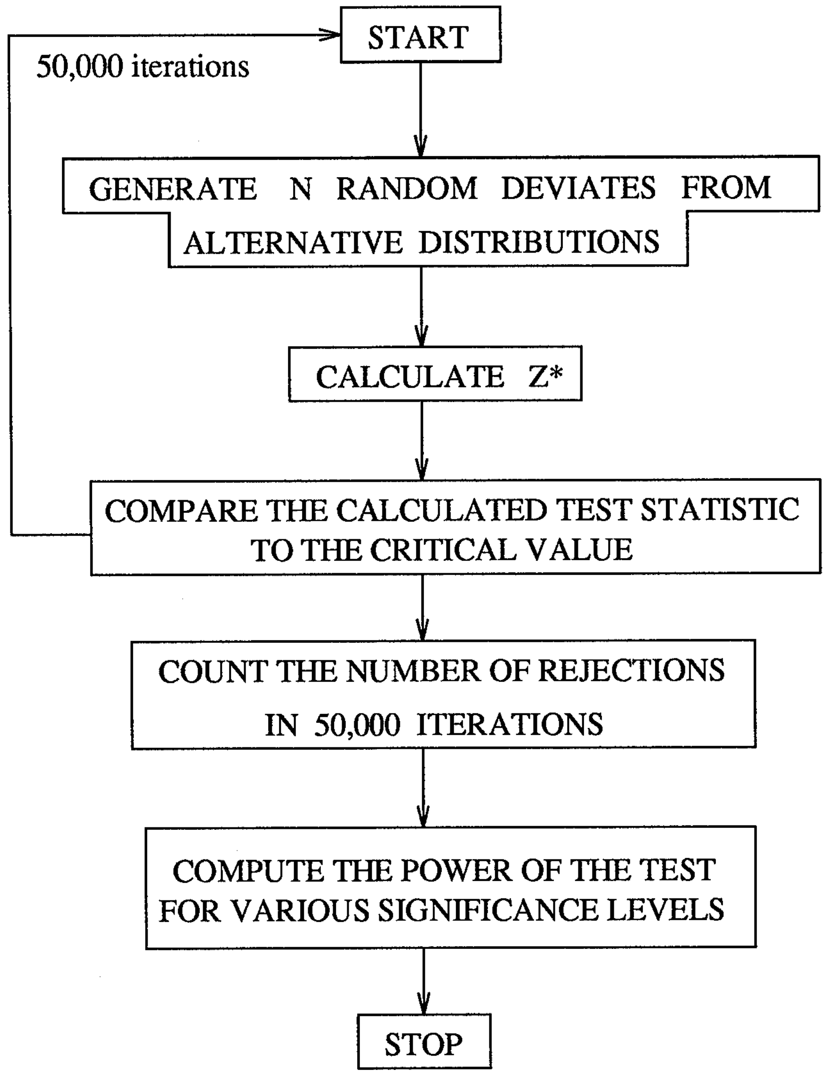

Figure 3.4 Generation of Power tables for the Z^* test statistic

IV. FINDINGS AND DISCUSSION OF ANALYSIS

This chapter discusses the findings of this research effort. Results applicable to each of the objectives set forth in Chapter 1, Introduction, are presented in sequence.

4.1 The Z* Test Statistic

The Z^* statistic was utilized during all phases of this thesis effort. On the whole, it was relatively simple to compute once the the differences between the expected values of the order statistics required in the denominator of G_i were available. These values were taken directly from the tables listed in Harter (10).

4.2 Critical Values for the Z* Test Statistic

Critical values for Z^* were obtained via Monte Carlo method by averaging the percentiles obtained in 10 blocks of 10,000 iterations. Linear interpolation at the appropriate levels of significance yielded the desired numerical values of the critical values of the Z^*. The initial experiments included the sample sizes ranging from 5 to 35 observations. In order to verify the trend of the critical values as the sample size changed, an additional sample of size $n = 40$ was applied to the gamma distribution with shape parameter $\alpha = 0.5$. For illustration purposes, the critical values obtained for shape parameter $\alpha = 1.0$ are presented in Table 4.1.

Table 4.1 Critical values for Z^* test statistic: Sample size N, shape parameter 1.0

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	1.0	1.2918	1.3564	1.4388	1.5543	1.7426
10	1.0	1.1760	1.2160	1.2659	1.3390	1.4695
15	1.0	1.1356	1.1667	1.2059	1.2634	1.3676
20	1.0	1.1157	1.1424	1.1759	1.2248	1.3141
25	1.0	1.1012	1.1255	1.1549	1.1987	1.2781
30	1.0	1.0917	1.1128	1.1396	1.1780	1.2516
35	1.0	1.0853	1.1045	1.1287	1.1662	1.2347

Obviously, as α decreases from 0.20 to 0.01, the critical values increase as expected. Within each α level, the critical values decrease gradually as the sample size increases from 5 to 35. However, the rate of decrease within each α level is not linear. This trend was noted for all shape parameters considered in this thesis. Tabled values for all shape parameters taken from complete samples are listed in Appendix A. The critical values for Type II censored samples with a censoring level of 20% are listed in Appendix B.

4.3 Power Study of Z* Test Statistic

The power of a goodness-of-fit test is the probability of rejecting the null hypothesis, H_0, when it is false. Therefore, we seek power values as close to 1.0 as possible for any distribution that is not a gamma distribution with the specified shape parameter. Nine alternative distributions were considered in the power study of Z^*.

4.3.1 Cross-checks.

Two different cross-checks were used throughout the computation phases of critical values and power study:

- When the null hypothesis was input into the power study computer program, the attained power should have matched the claimed level of significance, α. In all cases, power attained when H_0 within 5 percent of the α level. So, the accuracy of the critical values found by the critical value Fortran program in Appendix E was checked by the results of power study Fortran program in Appendix F.
- Alternate distributions number 2, 3 and 4 were the gamma distributions with shape parameters equal to 1.5, 2.5 and 4.0. When the null hypothesis is true the power of the null hypothesis and the power of the gamma distribution with the same shape parameter should be equal. Therefore, the accuracy of the power values were checked by duplicating the distributions.

Power study results for the Z^* test statistic were mixed. For shape parameters less than two, the test yielded good to excellent results. For shape parameter two and higher, the test results were poor. Of particular interest was the excellent performance of the Z^* test statistic when the null hypothesis was more skewed than the alternative distributions. This observation implies that Z^* test statistic is very effective for certain alternatives and somewhat weak for others. This results indicate that the Z^* test statistic is a *directional test.*

4.4 Power Results for the Z* Test Statistic

In the following sections, the power results are discussed in four phases due to the differing levels of power attained by this goodness-of-fit test. As the null hypothesis H_0 changes from being highly skewed to symmetric, the power results change from excellent to poor. Of course, larger sample sizes produce better power at all values of the null hypothesis. The four groupings for discussion are:

- Gamma shape parameter 0.5
- Gamma shape parameter 1.0
- Gamma shape parameter 1.5
- Gamma shape parameters 2.0, 3.0, 4.0.

For classifying the power values, the following subjective scale is adopted:

- Power of the test values of 0.90 or greater are excellent
- Power of the test values of 0.60 to 0.89 are good
- Power of the test values of 0.40 to 0.59 are fair
- Power of the test values below 0.40 are poor

Although this scale is subjective, this study requires the classification of the powers for clarity and comparison.

4.4.1 **Power Results for Shape Parameter $\alpha = 0.5$ from Complete Samples.**

The gamma distribution with shape parameter 0.5 is the most skewed distribution considered in this analysis. At this shape parameter, the gamma distribution highly differs from all the alternatives except the lognormal distribution (0,2). For this reason, it may be expected that power should be high against every alternative distribution but the lognormal. This expectation is verified by the power values attained by the Monte Carlo experiment. All power tables from sample size 5 through 35 are given in Appendix C. Table 4.2 shows the power values for sample size $n = 10$.

Table 4.2 Power Study: Sample size 10, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.098	0.049	0.009
Gamma(1.5)	0.767	0.689	0.577	0.408	0.142
Gamma(2.5)	0.871	0.816	0.729	0.572	0.255
Gamma(4.0)	0.915	0.875	0.806	0.671	0.353
Weibull(2.0)	0.941	0.909	0.849	0.727	0.407
Weibull(3.0)	0.973	0.954	0.920	0.840	0.573
Lognormal(0,2)	0.058	0.041	0.026	0.011	0.002
Lognormal(1,1)	0.486	0.404	0.308	0.191	0.051
Beta(2,2)	0.974	0.955	0.917	0.825	0.535
Beta(2,3)	0.955	0.927	0.876	0.761	0.439

At a significance level of 0.05, the power of the test is at least good for 5 of the 9 alternate hypotheses H_a. For α levels of 0.10 and higher, the power ranges from fair to excellent. These were the highest power values attained for the Z^* test statistic for sample size $n = 10$..

As the sample size increases, the power of the test is excellent against all of the alternatives except lognormal distribution (0,2), which is the only alternative distribution more skewed than the hypothesized gamma distribution with shape $\alpha = 0.5$. The lognormal distribution (1,1) is rejected at fair to excellent level, since it is slightly less skewed than the gamma distribution with shape parameter $\alpha = 0.5$. Table 4.3 illustrates the effect of the sample size on the power values.

Table 4.3 Power Study: Sample size 35, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.153	0.103	0.051	0.011
Gamma(1.5)	1.000	0.999	0.998	0.992	0.943
Gamma(2.5)	1.000	1.000	1.000	0.999	0.993
Gamma(4.0)	1.000	1.000	1.000	1.000	0.998
Weibull(2.0)	1.000	1.000	1.000	1.000	0.999
Weibull(3.0)	1.000	1.000	1.000	1.000	1.000
Lognormal(0,2)	0.011	0.007	0.005	0.002	0.000
Lognormal(1,1)	0.941	0.918	0.877	0.792	0.562
Beta(2,2)	1.000	1.000	1.000	1.000	1.000
Beta(2,3)	1.000	1.000	1.000	1.000	1.000

These results imply that the Z^* test statistic is more powerful against the distributions which are more skewed than the gamma distribution with the specified shape parameter α stated in the null hypothesis.

4.4.2 Power Results for Shape Parameter $\alpha = 0.5$ from Censored Samples.

The power values obtained from censored samples were close to those obtained from complete samples. In most cases, the power against alternative distributions was slightly lower for the censored case due to the reduced information regarding the sample. Table 4.4 presents the power of the tests for the censored case where the sample size was 10, and the number of observations was $r = 8$.

Table 4.4 Power Study: Sample size 10, observations 8, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.151	0.102	0.049	0.010
Gamma(1.5)	0.658	0.575	0.465	0.303	0.091
Gamma(2.5)	0.765	0.694	0.594	0.423	0.154
Gamma(4.0)	0.824	0.761	0.670	0.509	0.216
Weibull(2.0)	0.848	0.789	0.701	0.539	0.231
Weibull(3.0)	0.908	0.865	0.798	0.662	0.353
Lognormal(0,2)	0.129	0.093	0.060	0.028	0.005
Lognormal(1,1)	0.507	0.423	0.326	0.196	0.051
Beta(2,2)	0.887	0.837	0.756	0.607	0.287
Beta(2,3)	0.852	0.794	0.705	0.542	0.230

At a significance level of 0.05, the power of the test is between fair and good for 6 of the 9 alternate hypotheses H_a. For α levels of 0.10 and higher, the power is good for most of the alternatives with the exception of the lognormal distribution.

The effect of the sample size may be observed in Table 4.5, which depicts the results of a power study of censored observations for a sample of $n = 30$ and $r = 24$ observations.

Table 4.5 Power Study: Sample size 30, observations 24, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.151	0.101	0.050	0.010
Gamma(1.5)	0.993	0.986	0.973	0.930	0.747
Gamma(2.5)	0.999	0.998	0.995	0.984	0.919
Gamma(4.0)	1.000	0.999	0.999	0.995	0.963
Weibull(2.0)	1.000	0.999	0.999	0.996	0.969
Weibull(3.0)	1.000	1.000	1.000	0.999	0.992
Lognormal(0,2)	0.117	0.085	0.054	0.026	0.005
Lognormal(1,1)	0.951	0.926	0.884	0.785	0.521
Beta(2,2)	1.000	1.000	1.000	0.999	0.984
Beta(2,3)	1.000	1.000	0.999	0.996	0.969

At a significance level of 0.05, the power is excellent for 7 of the 9 alternative hypotheses. The lognormal distribution (1,1) had a power of 0.785, while the other lognormal distribution (0,2) had a power 0.026.

4.4.3 **Power Results for Shape Parameter $\alpha = 1.0$ from Complete Samples.** The gamma distribution with shape parameter $\alpha = 1.0$ is a non-symmetric distribution. It is equivalent to the exponential distribution. As Figure 4.1 indicates, the gamma distribution with shape parameter $\alpha = 1.0$ is less skewed than the gamma distribution with shape parameter $\alpha = 0.5$.

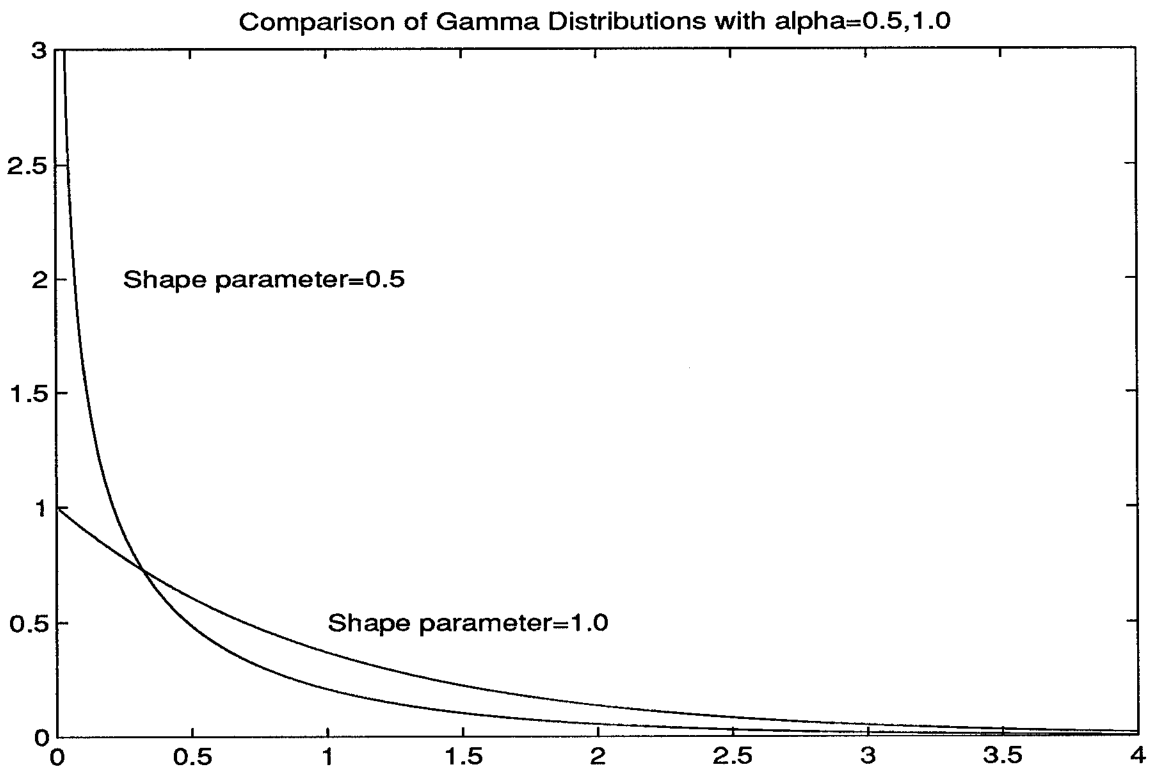

Figure 4.1 Gamma Distributions with $\alpha = 0.5$ and $\alpha = 1.0$.

The power results varied by significance level. For a significance level of 0.10 or higher, fair to good power was attained at sample sizes of 15 or greater except when the alternative distribution was the lognormal distribution.

Tables 4.6 and 4.7 present the power study results for sample sizes 10 and 35. All other power tables for this shape parameter are included in Appendix C.

Table 4.6 Power Study: Sample size 10, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.195	0.145	0.097	0.048	0.010
Gamma(1.5)	0.338	0.268	0.190	0.104	0.025
Gamma(2.5)	0.505	0.423	0.323	0.198	0.057
Gamma(4.0)	0.617	0.537	0.435	0.292	0.099
Weibull(2.0)	0.699	0.623	0.520	0.362	0.134
Weibull(3.0)	0.840	0.784	0.702	0.556	0.278
Lognormal(0,2)	0.009	0.006	0.003	0.001	0.000
Lognormal(1,1)	0.136	0.102	0.066	0.032	0.007
Beta(2,2)	0.844	0.786	0.698	0.544	0.253
Beta(2,3)	0.761	0.688	0.585	0.421	0.163

Table 4.7 Power Study: Sample size 35, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.151	0.101	0.050	0.009
Gamma(1.5)	0.639	0.564	0.462	0.311	0.110
Gamma(2.5)	0.933	0.904	0.853	0.747	0.476
Gamma(4.0)	0.984	0.974	0.957	0.912	0.748
Weibull(2.0)	0.996	0.992	0.985	0.964	0.861
Weibull(3.0)	1.000	1.000	0.999	0.997	0.984
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.087	0.064	0.042	0.020	0.004
Beta(2,2)	1.000	1.000	0.999	0.997	0.979
Beta(2,3)	0.999	0.998	0.996	0.986	0.920

4.4.4 Power Results for Shape Parameter $\alpha = 1.0$ from Censored Samples.

The power values obtained from censored samples were close to those obtained from complete samples. In most cases, the power against alternative distributions was slightly lower for the censored case due to the reduced information regarding the sample. Table 4.8 presents the power of the test for the censored case where the sample size was 10, and the number of observations was $r = 8$.

Table 4.9 presents the censored case where the number of observations in a sample size of $n = 35$ was $r = 28$.

Table 4.8 Power Study: Sample size 10, observations 8, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.050	0.010
Gamma(1.5)	0.307	0.240	0.167	0.090	0.019
Gamma(2.5)	0.426	0.347	0.259	0.151	0.038
Gamma(4.0)	0.511	0.432	0.338	0.211	0.060
Weibull(2.0)	0.551	0.469	0.369	0.235	0.071
Weibull(3.0)	0.683	0.610	0.513	0.364	0.138
Lognormal(0,2)	0.031	0.020	0.012	0.005	0.001
Lognormal(1,1)	0.191	0.145	0.098	0.050	0.009
Beta(2,2)	0.636	0.556	0.452	0.304	0.101
Beta(2,3)	0.561	0.479	0.376	0.240	0.073

Table 4.9 Power Study: Sample size 35, observations 28, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.099	0.050	0.010
Gamma(1.5)	0.564	0.481	0.382	0.250	0.081
Gamma(2.5)	0.862	0.813	0.737	0.609	0.331
Gamma(4.0)	0.946	0.923	0.883	0.800	0.568
Weibull(2.0)	0.964	0.946	0.914	0.843	0.613
Weibull(3.0)	0.995	0.992	0.986	0.968	0.885
Lognormal(0,2)	0.001	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.232	0.178	0.124	0.069	0.015
Beta(2,2)	0.989	0.982	0.966	0.927	0.771
Beta(2,3)	0.969	0.951	0.918	0.849	0.617

4.4.5 **Power Results for Shape Parameter $\alpha = 1.5$ from Complete Samples.** Fair to excellent power values were attained at high sample sizes of 30 and 35 for significance levels greater than or equal to 0.10 except when the alternative distributions were the lognormal distributions (0,2) and (1,1). At significance levels of 0.05 and 0.01, the test attained good results in only 3 of 9 alternate hypotheses. However, the sample sizes necessary for a satisfactory goodness-of-fit test are not prohibitively large. Table 4. 10 presents the power values for sample size $n = 10$. Table 4.11 presents the power values for sample size $n = 30$.

Table 4.10 Power Study: Sample size 10, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.147	0.098	0.048	0.009
Gamma(1.5)	0.199	0.150	0.099	0.050	0.010
Gamma(2.5)	0.331	0.262	0.185	0.100	0.023
Gamma(4.0)	0.444	0.366	0.273	0.165	0.045
Weibull(2.0)	0.538	0.453	0.349	0.214	0.061
Weibull(3.0)	0.725	0.651	0.550	0.397	0.161
Lognormal(0,2)	0.004	0.003	0.001	0.000	0.000
Lognormal(1,1)	0.070	0.050	0.030	0.014	0.002
Beta(2,2)	0.734	0.660	0.556	0.394	0.150
Beta(2,3)	0.615	0.530	0.419	0.267	0.084

Table 4.11 Power Study: Sample size 30, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.206	0.154	0.103	0.050	0.009
Gamma(1.5)	0.201	0.151	0.102	0.051	0.010
Gamma(2.5)	0.559	0.482	0.385	0.253	0.084
Gamma(4.0)	0.788	0.729	0.644	0.503	0.243
Weibull(2.0)	0.904	0.864	0.800	0.674	0.380
Weibull(3.0)	0.990	0.984	0.972	0.940	0.807
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.012	0.008	0.004	0.002	0.000
Beta(2,2)	0.992	0.986	0.975	0.940	0.786
Beta(2,3)	0.959	0.936	0.896	0.802	0.524

4.4.6 **Power Results for Shape Parameter $\alpha = 1.5$ from Censored Samples.** The power values obtained from censored samples for alternative distributions with the shape parameter $\alpha = 1.5$ were lower in most cases than the powers of complete samples. The two power values greater than complete samples were obtained against the lognormal (1,1) alternative distribution. The power values for censored samples are presented in Table 4.12 for sample size of $n = 10$ and in Table 4.13 for sample size of $n = 35$.

Table 4.12 Power Study: Sample size 10, observations 8, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.145	0.096	0.048	0.009
Gamma(1.5)	0.200	0.149	0.099	0.049	0.009
Gamma(2.5)	0.292	0.229	0.159	0.085	0.019
Gamma(4.0)	0.373	0.301	0.220	0.127	0.030
Weibull(2.0)	0.408	0.331	0.245	0.142	0.036
Weibull(3.0)	0.553	0.474	0.377	0.246	0.076
Lognormal(0,2)	0.018	0.011	0.006	0.002	0.000
Lognormal(1,1)	0.115	0.082	0.053	0.024	0.004
Beta(2,2)	0.505	0.425	0.325	0.200	0.057
Beta(2,3)	0.423	0.345	0.257	0.152	0.038

Table 4.13 Power Study: Sample size 35, observations 28, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.152	0.102	0.051	0.010
Gamma(1.5)	0.199	0.150	0.100	0.052	0.010
Gamma(2.5)	0.519	0.443	0.347	0.227	0.070
Gamma(4.0)	0.728	0.663	0.572	0.437	0.197
Weibull(2.0)	0.793	0.732	0.643	0.502	0.233
Weibull(3.0)	0.957	0.938	0.906	0.837	0.623
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.046	0.032	0.019	0.008	0.001
Beta(2,2)	0.911	0.875	0.813	0.702	0.414
Beta(2,3)	0.810	0.751	0.661	0.520	0.243

4.4.7 Power Results for Shape Parameters

α = 2.0, 3.0, and 4.0. As the shape parameter increased to 4.0, the Z^* test statistic did a poor job in discriminating between the null hypothesis H_0 and the alternate hypothesis H_a. As the shape parameter increased, the skewness of the alternative distributions relative to the hypothesized distribution induced lower power in the test statistic. When the sample size increased to 35 for shape parameter=4.0, the power to discriminate against the beta(2,2) and Weibull(3.0) alternative distributions was good at significance level of 0.10 or more. The power tables shown in Tables 4.14 - 4.16 were generated for shape parameters α =2.0, 3.0 and 4.0 and a sample size of $n = 35$, where the test attains higher power.

Table 4.14 Power Study: Sample size 35, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.150	0.100	0.051	0.010
Gamma(1.5)	0.076	0.052	0.031	0.013	0.002
Gamma(2.5)	0.337	0.268	0.194	0.110	0.027
Gamma(4.0)	0.621	0.545	0.451	0.313	0.118
Weibull(2.0)	0.806	0.742	0.650	0.501	0.227
Weibull(3.0)	0.981	0.970	0.950	0.902	0.730
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.002	0.001	0.000	0.000	0.000
Beta(2,2)	0.986	0.977	0.957	0.907	0.707
Beta(2,3)	0.921	0.882	0.816	0.690	0.383

Table 4.15 Power Study: Sample size 35, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.196	0.148	0.100	0.050	0.010
Gamma(1.5)	0.020	0.012	0.006	0.002	0.000
Gamma(2.5)	0.127	0.092	0.058	0.026	0.005
Gamma(4.0)	0.327	0.262	0.189	0.105	0.028
Weibull(2.0)	0.535	0.456	0.357	0.219	0.069
Weibull(3.0)	0.921	0.888	0.834	0.722	0.460
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.000	0.000	0.000	0.000	0.000
Beta(2,2)	0.938	0.907	0.853	0.733	0.448
Beta(2,3)	0.748	0.672	0.567	0.397	0.156

Table 4.16 Power Study: Sample size 35, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.149	0.098	0.049	0.010
Gamma(1.5)	0.008	0.005	0.003	0.001	0.000
Gamma(2.5)	0.065	0.044	0.025	0.010	0.002
Gamma(4.0)	0.200	0.152	0.101	0.051	0.010
Weibull(2.0)	0.376	0.302	0.215	0.118	0.028
Weibull(3.0)	0.841	0.790	0.710	0.567	0.292
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.000	0.000	0.000	0.000	0.000
Beta(2,2)	0.880	0.828	0.746	0.596	0.292
Beta(2,3)	0.606	0.517	0.403	0.251	0.073

4.4.8 **Power Results for Shape Parameter α =2, 3, and 4 from Censored Samples.** The power values obtained from censored samples were very low, and the Z^* test statistic was not useful. For shape parameters 2,3, and 4, the Z^* test statistic is weak even for complete samples. When censoring was added, the resulting power was lower. Table 4.17 presents the power values obtained for shape parameter $\alpha = 2.0$. The power values for Weibull(3.0) and beta (2,2) alternative distributions are at least good for significance levels of 0.10 or higher.

Table 4.17 Power Study: Sample size 35, observations 28, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.152	0.101	0.051	0.009
Gamma(1.5)	0.092	0.064	0.039	0.017	0.003
Gamma(2.5)	0.312	0.245	0.175	0.097	0.023
Gamma(4.0)	0.526	0.450	0.357	0.237	0.079
Weibull(2.0)	0.605	0.525	0.425	0.291	0.104
Weibull(3.0)	0.889	0.851	0.791	0.679	0.413
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.016	0.010	0.006	0.002	0.000
Beta(2,2)	0.796	0.733	0.643	0.497	0.229
Beta(2,3)	0.633	0.554	0.454	0.310	0.111

Table 4.18 presents the power values for the shape parameter $\alpha = 3.0$. At the significance level of 0.10 or more, the power value for Weibull(3.0) and beta(2,2) are fair to good.

Table 4.18 Power Study: Sample size 35, observations 28, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.152	0.102	0.051	0.010
Gamma(1.5)	0.034	0.022	0.011	0.004	0.000
Gamma(2.5)	0.142	0.103	0.065	0.031	0.005
Gamma(4.0)	0.301	0.238	0.170	0.094	0.022
Weibull(2.0)	0.369	0.299	0.220	0.126	0.031
Weibull(3.0)	0.742	0.678	0.589	0.446	0.203
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.005	0.003	0.002	0.001	0.000
Beta(2,2)	0.596	0.513	0.413	0.273	0.089
Beta(2,3)	0.402	0.326	0.241	0.142	0.035

Table 4.19 presents the power values obtained for the shape parameter $\alpha = 4.0$. The power for Weibull(3.0) alternative distribution are fair for significance levels of 0.10 or higher.

Table 4.19 Power Study: Sample size 35, observations 28, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.151	0.100	0.050	0.010
Gamma(1.5)	0.018	0.011	0.006	0.002	0.000
Gamma(2.5)	0.086	0.059	0.036	0.015	0.002
Gamma(4.0)	0.202	0.153	0.102	0.050	0.011
Weibull(2.0)	0.260	0.201	0.137	0.070	0.014
Weibull(3.0)	0.629	0.554	0.455	0.316	0.119
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.003	0.001	0.001	0.000	0.000
Beta(2,2)	0.472	0.394	0.298	0.179	0.051
Beta(2,3)	0.289	0.224	0.155	0.081	0.017

4.5 Comparison of Z* and its Competitors

Power data for the prominent goodness-of-fit test for the gamma distribution was available in Viviano (33). The power values listed by Viviano were limited to sample sizes of 5, 15 and 25 at alpha levels of 0.05 and 0.01 for only two shape parameters ($\alpha = 1.5$ and $\alpha = 4.0$). The nine alternate hypotheses used in this thesis were chosen to coincide with the alternative distributions used by Viviano to facilitate a direct comparison. Since alternative distributions #2, #3 and #4 in this research were essentially equivalent to the null hypothesis at the corresponding shape parameters, the power of the test for those distributions should be the significance level of the test. Therefore, subsequent analysis will involve only 6 alternate hypotheses, H_a. The three competing test statistics were:

- Kolmogorov-Smirnov (K-S)
- Cramer von Mises (W^2)
- Anderson-Darling (A^2)

4.5.1 Comparison Between Tests for Shape

Parameter α = 1.5. Tables 4. 20 and 4. 21 present the comparison of the competing test statistics. Although not very high, the power of the Z^* test statistic is higher than its competitors in 12 out of 14 direct comparisons for shape parameter $\alpha = 1.5$, involving the alternate hypotheses and two significance levels.

Table 4.20 Comparative Power Study of Z^* against other test statistics for sample size $n = 5$, shape $\alpha = 1.5$ and significance level= 0.05

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.050	0.051	0.047	0.049	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.051	0.051	0.047	0.049	Z^*
Gamma(2.5)	0.061	0.044	0.036	0.030	Z^*
Gamma(4.0)	0.071	0.042	0.034	0.026	Z^*
Weibull(2.0)	0.078	0.034	0.028	0.017	Z^*
Weibull(3.0)	0.099	0.040	0.030	0.015	Z^*
Lognormal(0,2)	0.019	0.383	0.405	0.423	A^2
Beta(2,2)	0.095	0.045	0.034	0.017	Z^*

Table 4.21 Comparative Power Study of Z^* against other test statistics for sample size $n = 5$, shape $\alpha = 1.5$ and significance level= 0.01

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.010	0.010	0.013	0.011	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.010	0.010	0.013	0.011	Z^*
Gamma(2.5)	0.014	0.007	0.007	0.006	Z^*
Gamma(4.0)	0.014	0.006	0.006	0.005	Z^*
Weibull(2.0)	0.015	0.003	0.002	0.002	Z^*
Weibull(3.0)	0.020	0.003	0.001	0.001	Z^*
Lognormal(0,2)	0.004	0.213	0.244	0.260	A^2
Beta(2,2)	0.021	0.003	0.002	0.001	Z^*

Tables 4.22 and 4.23 present the power values of the competing test statistics for the shape parameter $\alpha = 1.5$, **sample size 15** and alpha levels of 0.05 and 0.01. Z^* exceeds the power of its competitors in 12 of 14 direct comparisons.

Table 4.22 Comparative Power Study of Z^* against other test statistics for sample size $n = 15$, shape $\alpha = 1.5$ and significance level= 0.05

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.050	0.045	0.047	0.044	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.050	0.045	0.047	0.044	Z^*
Gamma(2.5)	0.140	0.053	0.056	0.037	Z^*
Gamma(4.0)	0.247	0.093	0.101	0.069	Z^*
Weibull(2.0)	0.345	0.123	0.145	0.093	Z^*
Weibull(3.0)	0.632	0.290	0.351	0.280	Z^*
Lognormal(0,2)	0.000	0.873	0.909	0.928	A^2
Beta(2,2)	0.622	0.245	0.304	0.234	Z^*

Table 4.23 Comparative Power Study of Z^* against other test statistics for sample size $n = 15$, shape $\alpha = 1.5$ and significance level= 0.01

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.011	0.007	0.008	0.009	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.011	0.007	0.008	0.009	Z^*
Gamma(2.5)	0.039	0.009	0.009	0.004	Z^*
Gamma(4.0)	0.086	0.022	0.024	0.011	Z^*
Weibull(2.0)	0.132	0.033	0.036	0.020	Z^*
Weibull(3.0)	0.362	0.106	0.142	0.086	Z^*
Lognormal(0,2)	0.000	0.748	0.813	0.844	A^2
Beta(2,2)	0.331	0.086	0.108	0.066	Z^*

4.5.2 Comparison Between Tests for Shape

Parameter $\alpha = 4.0$. Tables 4.24 and 4.25 present the power values of the Z^* test statistic and its competitors for the shape parameter $\alpha = 4.0$, **sample size 5** and significance levels of 0.05 and 0.01. Although not particularly high, the power values of Z^* are higher than its competitors in 8 out of 14 direct comparisons.

Table 4.24 Comparative Power Study of Z^* against other test statistics for sample size $n = 5$, shape $\alpha = 4.0$ and significance level= 0.05

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.051	0.057	0.055	0.054	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.027	0.075	0.076	0.078	A^2
Gamma(2.5)	0.039	0.061	0.061	0.063	A^2
Gamma(4.0)	0.049	0.057	0.055	0.054	Z^*
Weibull(2.0)	0.058	0.047	0.045	0.042	Z^*
Weibull(3.0)	0.093	0.050	0.050	0.040	Z^*
Lognormal(0,2)	0.004	0.439	0.462	0.489	A^2
Beta(2,2)	0.097	0.055	0.054	0.044	Z^*

Table 4.25 Comparative Power Study of Z^* against other test statistics for sample size $n = 5$, shape $\alpha = 4.0$ and significance level= 0.01

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.009	0.014	0.013	0.012	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.005	0.018	0.019	0.024	A^2
Gamma(2.5)	0.007	0.015	0.014	0.016	A^2
Gamma(4.0)	0.009	0.014	0.013	0.012	Z^*
Weibull(2.0)	0.011	0.011	0.010	0.007	Z^*
Weibull(3.0)	0.019	0.011	0.009	0.007	Z^*
Lognormal(0,2)	0.001	0.269	0.306	0.340	A^2
Beta(2,2)	0.021	0.008	0.008	0.006	Z^*

Tables 4. 26 and 4. 27 present the power of the competing test statistics.For the shape parameter $\alpha = 4.0$, **sample size 15**, and alpha levels of 0.05 and 0.01, Z^* exceeds the power of its competitors in 8 of 14 direct comparisons. comparison between competing test statistics.

Table 4.26 Comparative Power Study of Z^* against other test statistics for sample size $n = 15$, shape $\alpha = 4.0$ and significance level= 0.05

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.049	0.052	0.051	0.045	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.007	0.117	0.135	0.147	A^2
Gamma(2.5)	0.022	0.060	0.062	0.064	A^2
Gamma(4.0)	0.049	0.052	0.051	0.045	Z^*
Weibull(2.0)	0.081	0.054	0.057	0.050	Z^*
Weibull(3.0)	0.263	0.116	0.133	0.119	Z^*
Lognormal(0,2)	0.000	0.915	0.949	0.963	A^2
Beta(2,2)	0.266	0.112	0.131	0.117	Z^*

Table 4.27 Comparative Power Study of Z^* against other test statistics for sample size $n = 15$, shape $\alpha = 4.0$ and significance level= 0.01

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.010	0.010	0.010	0.009	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.001	0.034	0.040	0.049	A^2
Gamma(2.5)	0.004	0.013	0.015	0.017	A^2
Gamma(4.0)	0.009	0.010	0.010	0.009	Z^*
Weibull(2.0)	0.017	0.012	0.013	0.010	Z^*
Weibull(3.0)	0.089	0.036	0.043	0.037	Z^*
Lognormal(0,2)	0.000	0.817	0.883	0.907	A^2
Beta(2,2)	0.081	0.034	0.040	0.034	Z^*

Tables 4. 28 and 4. 29 present the power comparison between the competing test statistics for the shape parameter $\alpha = 4.0$, **sample size 25** and alpha levels of 0.05 and 0.01. Z^* test statistic exceeds that of the competition in 8 of 14 direct comparisons.

Table 4.28 Comparative Power Study of Z^* against other test statistics for sample size $n = 25$, shape $\alpha = 4.0$ and significance level= 0.05

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.048	0.052	0.051	0.048	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.002	0.177	0.202	0.226	A^2
Gamma(2.5)	0.014	0.072	0.075	0.075	A^2
Gamma(4.0)	0.047	0.052	0.051	0.048	Z^*
Weibull(2.0)	0.098	0.063	0.063	0.054	Z^*
Weibull(3.0)	0.423	0.198	0.230	0.218	Z^*
Lognormal(0,2)	0.000	0.993	0.997	0.999	A^2
Beta(2,2)	0.439	0.058	0.211	0.204	Z^*

Table 4.29 Comparative Power Study of Z^* against other test statistics for sample size $n = 25$, shape $\alpha = 4.0$ and significance level= 0.01

Distribution	Z^*	K-S	W^2	A^2	Best power by
Original, H_0	0.010	0.010	0.009	0.008	$Z^*, K-S, W^2, A^2$
Gamma(1.5)	0.000	0.057	0.066	0.070	A^2
Gamma(2.5)	0.002	0.017	0.016	0.017	A^2
Gamma(4.0)	0.010	0.010	0.009	0.008	Z^*
Weibull(2.0)	0.023	0.012	0.011	0.008	Z^*
Weibull(3.0)	0.180	0.061	0.082	0.074	Z^*
Lognormal(0,2)	0.000	0.968	0.985	0.991	A^2
Beta(2,2)	0.176	0.051	0.063	0.054	Z^*

4.6 Power Study Results that Supports the Idea that Z* is a Directional Test

In order to support the discussion of Z^* test statistic as a directional test, Weibull distributions with shape parameters ranging from 0.5 to 6.0 were tested. Power values of the Z^* test statistic were observed as the shape of a particular alternative distribution changes. Since Weibull distribution has a variety of shapes ranging from a highly skewed distribution to symmetric distribution, it was selected as the alternative distribution. Figure 4.2 presents the the changes in the plot of Weibull distribution as the shape parameter changes from 0.5 to 6.0, compared to the gamma distribution (1.0). Figure 4.2 shows that gamma distribution with shape

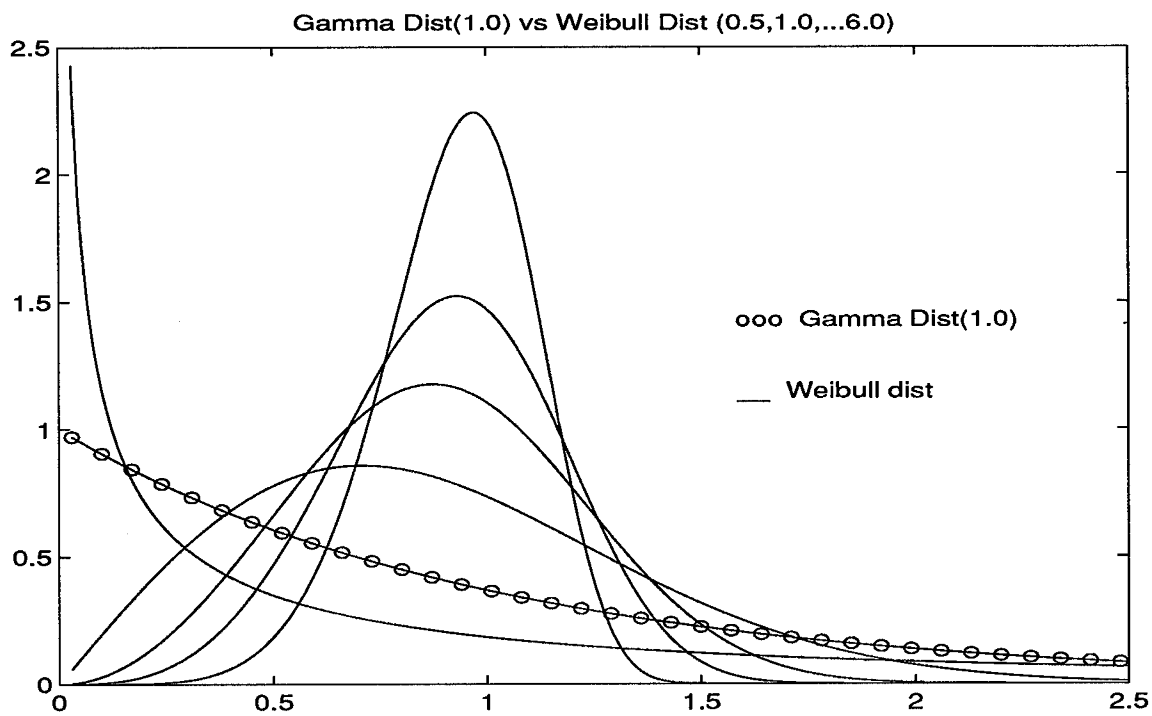

Figure 4.2 Gamma(1.0) vs Weibull family

parameter $\alpha = 1.0$ is more skewed than most of the Weibull distribution presented in the figure. Only Weibull distribution with shape parameter $k = 0.5$ was more skewed than the gamma(1.0).

Table 4.30 present the power values of Z^* test against Weibull alternatives with shape parameters $k = 0.5, 1.0, \ldots, 6.0$. At a sample size of 15, power values against Weibull alternatives are excellent for $k = 2.0, 3.0, 4.0$, and 6.0. The power value for Weibull (0.5) is poor, and the power value for Weibull (1.0) is the specified significance level, since Weibull (1.0) and the gamma (1.0) have the same probability density function, e^{-x}.

Table 4.30 Power Study: Sample size 15, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
H_o	0.199	0.149	0.100	0.051	0.010
Weibull(0.5)	0.002	0.001	0.001	0.000	0.000
Weibull(1.0)	0.200	0.151	0.100	0.049	0.010
Weibull(2.0)	0.865	0.812	0.731	0.590	0.301
Weibull(3.0)	0.955	0.933	0.895	0.810	0.575
Weibull(4.0)	0.975	0.961	0.935	0.878	0.689
Weibull(6.0)	0.986	0.977	0.960	0.921	0.780

At the sample size of 35, the power values for less skewed alternatives were excellent as presented in Table 4.31. The power values for more skewed Weibull (0.5) alternative were very poor at all significance levels.

Table 4.31 Power Study: Sample size 35, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
H_o	0.197	0.149	0.100	0.049	0.010
Weibull(0.5)	0.000	0.000	0.000	0.000	0.000
Weibull(1.0)	0.200	0.150	0.102	0.050	0.009
Weibull(2.0)	0.996	0.993	0.986	0.965	0.859
Weibull(3.0)	1.000	1.000	0.999	0.997	0.984
Weibull(4.0)	1.000	1.000	1.000	0.999	0.995
Weibull(6.0)	1.000	1.000	1.000	1.000	0.998

Figure 4.3 shows that half of the Weibull distributions presented are less skewed than the gamma distribution with shape parameter $\alpha = 2.5$. Weibull distributions with shape parameters $k = 05, 1.0, 2.0$ are more skewed than the gamma (2.5) while the other Weibull distributions with $k = 3.0, 4.0, 6.0$ are less skewed.

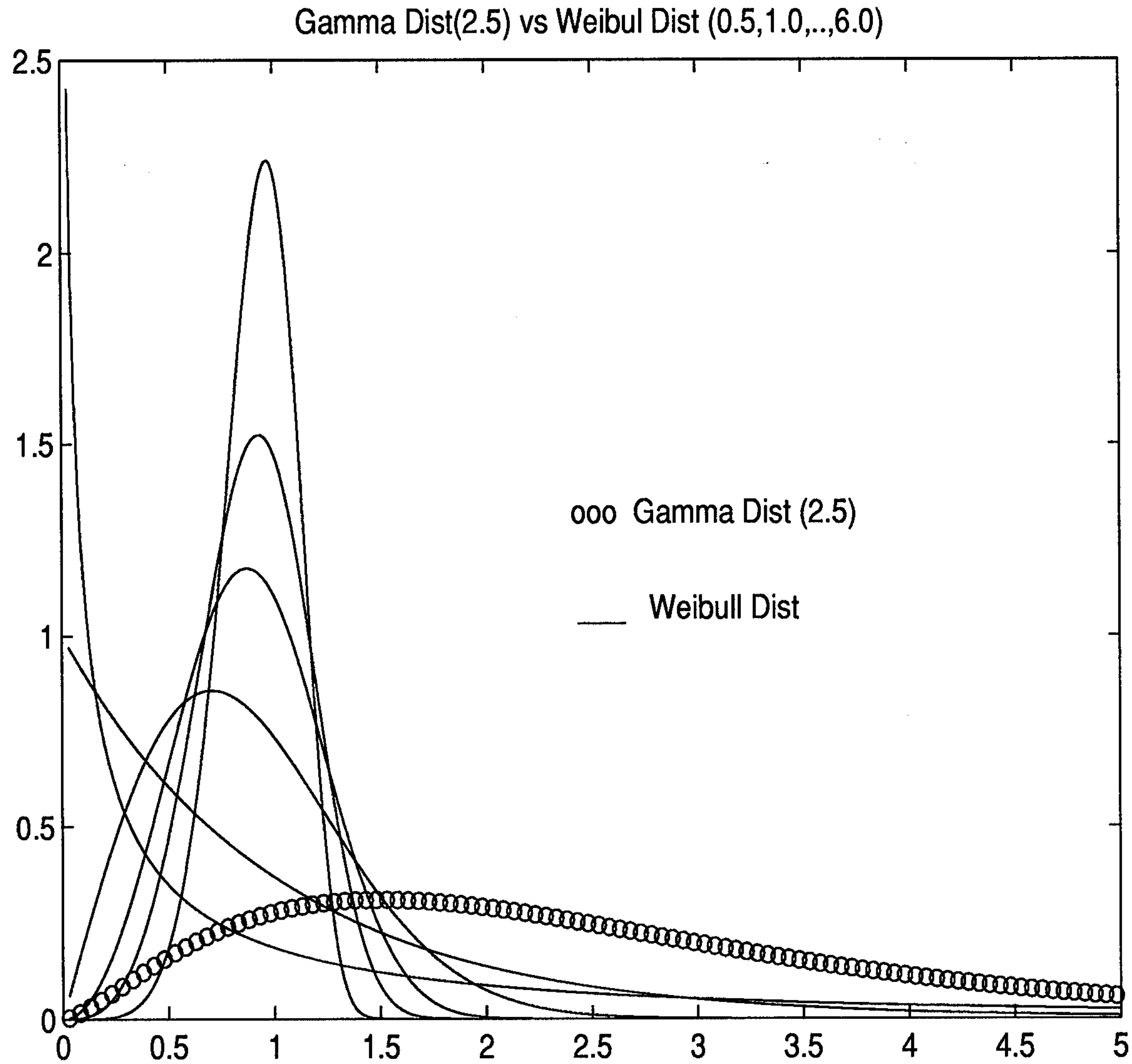

Figure 4.3 Gamma (2.5) vs Weibull family

Table 4.32 presents the power values for those Weibull alternative distributions. It was observed that the power values for less skewed Weibull alternative distributions were good at a sample size of 15, and excellent at a sample size of 35.

Table 4.32 Power Study: Sample size 15, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
H_o	0.197	0.148	0.098	0.048	0.009
Weibull(0.5)	0.000	0.000	0.000	0.000	0.000
Weibull(1.0)	0.031	0.020	0.012	0.004	0.001
Weibull(2.0)	0.443	0.363	0.268	0.155	0.041
Weibull(3.0)	0.719	0.649	0.550	0.400	0.169
Weibull(4.0)	0.820	0.767	0.686	0.551	0.289
Weibull(6.0)	0.889	0.849	0.788	0.675	0.423

Table 4.33 presents the power values at a sample size of 35. The power values for more skewed alternatives were poor, while the power for less skewed alternatives were good to excellent.

Table 4.33 Power Study: Sample size 35, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
H_o	0.201	0.151	0.099	0.050	0.010
Weibull(0.5)	0.000	0.000	0.000	0.000	0.000
Weibull(1.0)	0.004	0.002	0.001	0.000	0.000
Weibull(2.0)	0.661	0.582	0.478	0.328	0.116
Weibull(3.0)	0.956	0.934	0.895	0.814	0.575
Weibull(4.0)	0.987	0.980	0.965	0.929	0.794
Weibull(6.0)	0.996	0.994	0.988	0.976	0.913

4.7 Relationship between the Critical Values and the Sample Size

The analyses conducted throughout this research effort were done at sample sizes in multiples of five. Thus, critical values were only tabulated at these multiples. In order to extend the applicability of this goodness-of-fit test, a relationship between critical value and sample size, shape parameter, censoring % and significance level was investigated.

Figure 4.4 presents the relation between the critical values and the shape parameter for a sample size of 5 and a significance level of 0.05.

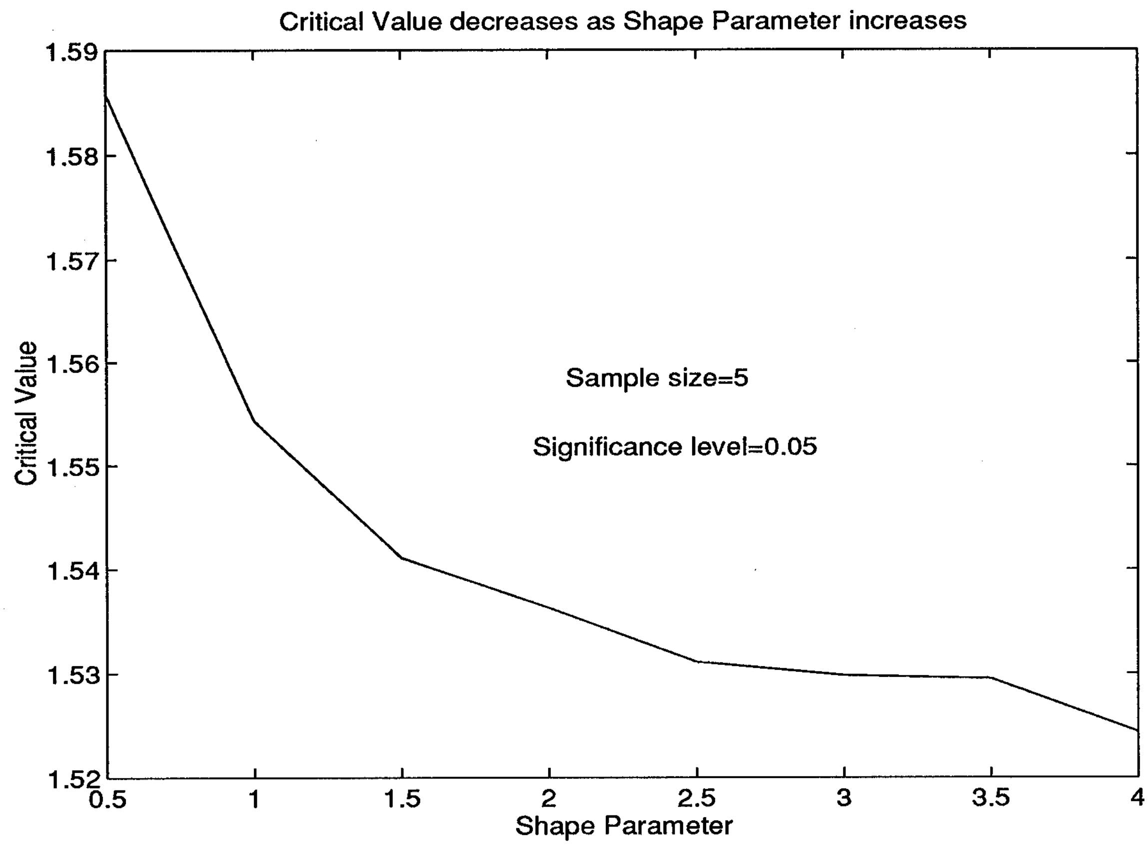

Figure 4.4 The relation between the Critical Values and the Shape parameter

Figure 4.5 presents the relation between the critical values and the sample size for shape parameter 0.5 and significance level 0.20.

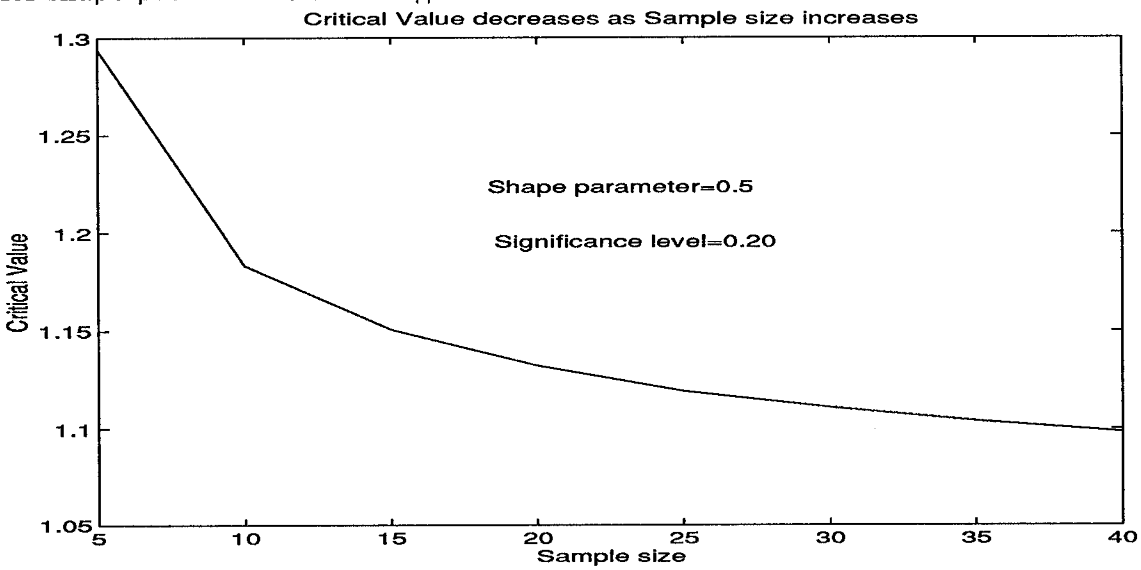

Figure 4.5 The relation between the Critical Values and the Sample size.

Figure 4.6 presents the relation between the critical values and the significance level for a sample size of 40 and shape parameter 0.5.

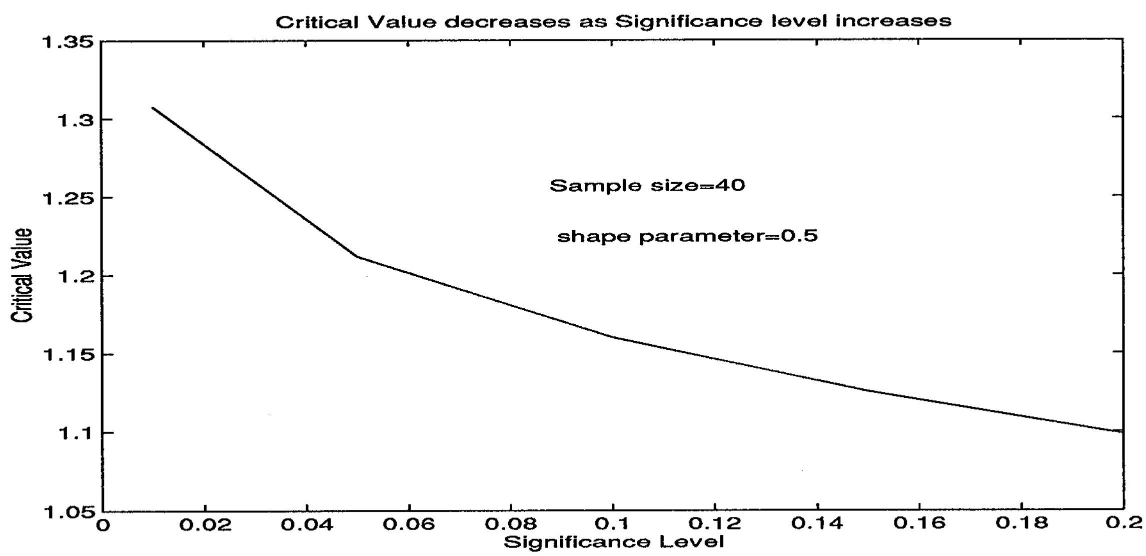

Figure 4.6 The relation between the Critical Values and the Significance level

A regression analysis was conducted to estimate the critical values for the Z^* test as a function of the sample size , gamma shape parameter, significance level, and degree of censoring.The resulting regression function is the linear combination of the variables and the coefficients given in Table 4.33, and is expressed as

$$Critical\ Value = Constant + a_1\, Censor\,\% + a_2\, Censor\,\% * Sample\ size +$$
$$a_3\,(Censor\,\% * Significance\ level) + \ldots + a_7 \log(Significance\ level)$$

This regression function can be used for **both censored and complete samples** to calculate critical values.

Table 4.34 Regression function representing the relation between the critical values and sample parameters **for complete or censored samples.**

Variables	Coefficients
$Constant$	2.77599
$Censor\,\%$ $(80, 90, \ldots, 100)$	-0.00905
$Censor\,\%*Sample\ size$	0.00004671
$Censor\,\%*Significance\ level$	-0.01238
$Sample\ size*Significance\ level$	0.04137
$\log(Sample\ size)$	-0.67283
$\log(Shape\ parameter)$	-0.05469
$\log(Significance\ level)$	-0.12667

R^2	0.9803
MSE	0.00055

Table 4.35 presents the coefficients of the regression function prepared exclusively for the **complete samples**.

Table 4.35 Regression functions representing the relation between the critical values and sample parameters **for complete samples only**.

Variables	Coefficients
$Constant$	1.89375
$Sample\ size$	0.00457
$Sample\ size * Significance\ level$	0.04359
$\log(Sample\ size)$	-0.69377
$\log(Shapeparameter)$	-0.06450
$\log(Significance\ level)$	-0.12673
$\sin(Significance\ level)$	-1.21745

R^2	0.9719

Table 4.36 presents the coefficients of several regression functions prepared for complete samples and various shape parameters.

Table 4.36 Regression functions representing the relation between the critical values and sample parameters **for different shape parameters**.

Variable	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 1.5$	$\alpha = 2.0$
$Constant$	1.76451	1.88435	1.80714	1.80620
n	0	0.00428	0	0
$n * sig$	0.04483	0.04422	0.05101	0.05046
$\log(n)$	-0.50987	-0.69058	-0.56634	-0.56432
$\log(sig)$	-0.16220	-0.13079	-0.12581	-0.12047
$\sin(sig)$	-1.38730	-1.25080	-1.34907	-1.33624
R^2	0.98450	0.98180	0.97290	0.97210

Variable	$\alpha = 2.5$	$\alpha = 3.0$	$\alpha = 3.5$	$\alpha = 4.0$
$Constant$	1.80077	1.80316	1.80263	1.79663
$n * sig$	0.05018	0.05066	0.05129	0.05088
$\log(n)$	-0.56037	-0.56334	-0.56461	-0.56109
$\log(sig)$	-0.11844	-0.11802	-0.11848	-0.11881
$\sin(sig)$	-1.32746	-1.33122	-1.33869	-1.32586
R^2	0.97190	0.97050	0.96820	0.96810

Further analysis on the regression was conducted to show that regression functions represent the sample of the critical values within a determined range. The regression function developed for complete and censored samples was examined. Figure 4.7 indicates that the predicted values by the regression is close to the critical values. The line represents the ideal case which predicted values are the same as the critical values. This regression function is valid for

- Censoring between 80% and 100%. 95 would be used in the regression function to represent 5% Type II censoring.
- Shape parameters between 0.5 and 4.0.
- Sample sizes between 5 and 35.
- Significance levels between 0.20 and 0.01.

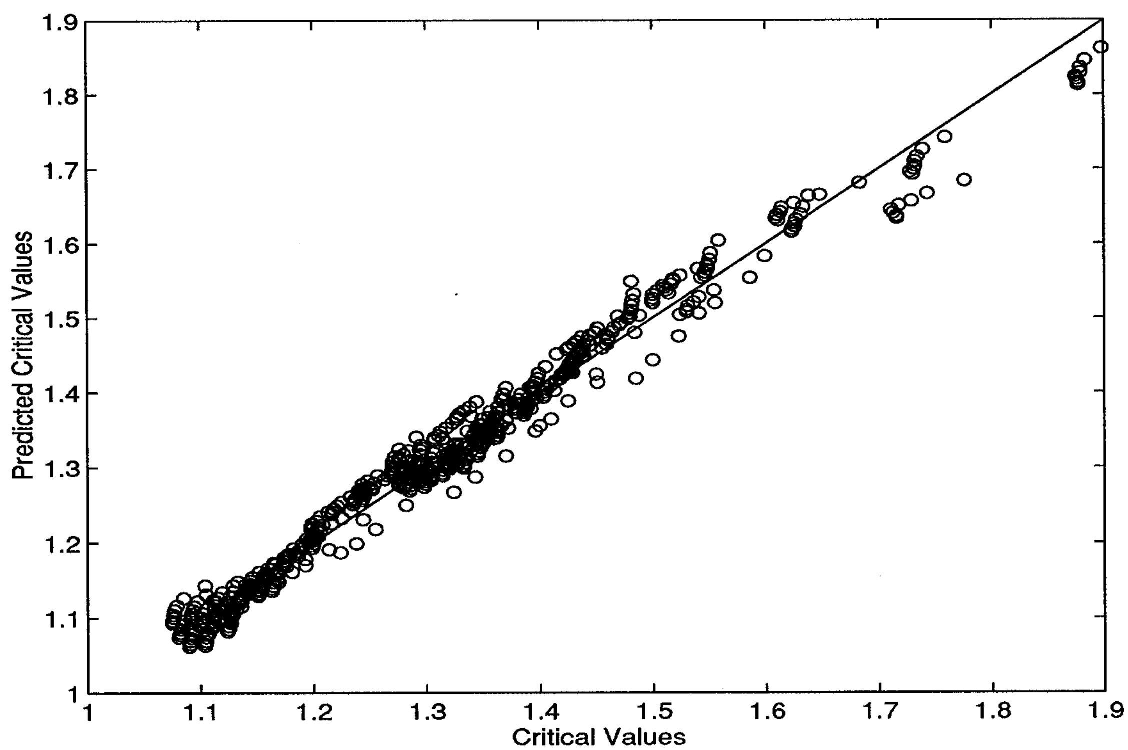

Figure 4.7 The Relation between the Predicted Values and the Critical values

More detailed analysis of this regression was presented in Appendix G.

Similar analysis was conducted for complete samples and shape parameter $\alpha = 0.5$. Figure 4.8 indicates that the predicted values were close to the critical values. The line represents the ideal case which predicted values are the same as the critical values. This regression function is valid for

- Shape parameter 0.5 only.
- Complete samples only.
- Sample sizes between 5 and 35.
- Significance levels between 0.20 and 0.01.

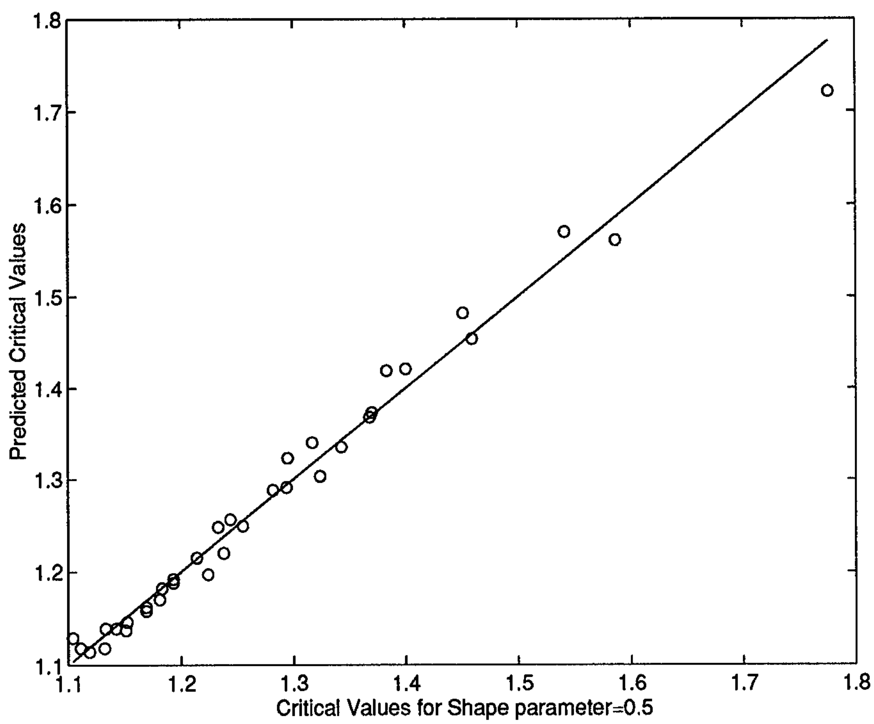

Figure 4.8 The approximity of Predicted Values to the Critical values for Shape Parameter $\alpha = 0.5$

More detailed analysis of this regression function was presented in Appendix H.

V. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations are based on the findings obtained during this research effort.

5.1 Conclusions

The following conclusions are made based on the critical value and power study programs accomplished during this thesis:

1. The tabled critical values of Z^* for the gamma distribution with shape parameter known are valid. In the Monte Carlo simulations, the goodness-of-fit test achieved the claimed level of significance when the null hypothesis is true.

2. The power of the Z^* test statistic is poor when the alternative distribution, H_a, is more skewed than the gamma distribution with specified shape parameter.

3. The power of the Z^* test statistic is good to excellent in the situations where the null hypothesis, H_0, is more skewed than the alternative distributions:

4. For shape parameter $\alpha = 1.0$, Z^*, achieves more power than any other current test statistic used in this research for the alternative distributions studied except lognormal alternatives.

5. Critical values for Z^* at sample sizes not explicitly computed in this thesis can be approximated with the regression. High values of R^2 were attained with regression functions for each shape parameter and for the case when all parameters are included in the regression.

5.2 Recommendations

The following recommendations are proposed for further study:

A sequential test using the Z^* and A-D test statistics as a couple should be applied to the gamma distribution. Since these two test statistics are the most powerful at different conditions, combining their powers by a sequential test should increase the overall power.

A further study of a modified sequential test using the Z^* and A-D test statistics and a sample estimate of the shape parameter is highly recommended.

Appendix A. Critical Value Tables for the Z* Test Statistic from Complete Samples

Table A.1 Critical values for Z^* test statistic: Sample size N, shape parameter 0.5

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	0.5	1.2939	1.3681	1.4588	1.5858	1.7757
10	0.5	1.1833	1.2331	1.2952	1.3827	1.5407
15	0.5	1.1507	1.1929	1.2442	1.3168	1.4508
20	0.5	1.1323	1.1688	1.2142	1.2816	1.4000
25	0.5	1.1192	1.1517	1.1932	1.2553	1.3698
30	0.5	1.1109	1.1419	1.1805	1.2380	1.3427
35	0.5	1.1039	1.1327	1.1693	1.2239	1.3238
40	0.5	1.0984	1.1254	1.1600	1.2118	1.3077

Table A.2 Critical values for Z^* test statistic: Sample size N, shape parameter 1.0

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	1.0	1.2918	1.3564	1.4388	1.5543	1.7426
10	1.0	1.1760	1.2160	1.2659	1.3390	1.4695
15	1.0	1.1356	1.1667	1.2059	1.2634	1.3676
20	1.0	1.1157	1.1424	1.1759	1.2248	1.3141
25	1.0	1.1012	1.1255	1.1549	1.1987	1.2781
30	1.0	1.0917	1.1128	1.1396	1.1780	1.2516
35	1.0	1.0853	1.1045	1.1287	1.1662	1.2347

Table A.3 Critical values for Z^* test statistic: Sample size N, shape parameter 1.5

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	1.5	1.2874	1.3488	1.4265	1.5411	1.7291
10	1.5	1.1686	1.2064	1.2539	1.3235	1.4487
15	1.5	1.1295	1.1581	1.1945	1.2484	1.3437
20	1.5	1.1085	1.1325	1.1626	1.2078	1.2948
25	1.5	1.0952	1.1163	1.1428	1.1823	1.2566
30	1.5	1.0859	1.1051	1.1289	1.1650	1.2330
35	1.5	1.0789	1.0968	1.1191	1.1522	1.2130

Table A.4 Critical values for Z^* test statistic: Sample size N, shape parameter 2.0

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	2.0	1.2855	1.3478	1.4239	1.5363	1.7183
10	2.0	1.1676	1.2039	1.2507	1.3170	1.4369
15	2.0	1.1274	1.1555	1.1903	1.2413	1.3376
20	2.0	1.1066	1.1302	1.1590	1.2026	1.2839
25	2.0	1.0925	1.1137	1.1399	1.1777	1.2479
30	2.0	1.0837	1.1023	1.1256	1.1600	1.2251
35	2.0	1.0765	1.0938	1.1150	1.1462	1.2058

Table A.5 Critical values for Z^* test statistic: Sample size N, shape parameter 2.5

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	2.5	1.2834	1.3442	1.4213	1.5311	1.7106
10	2.5	1.1652	1.2010	1.2459	1.3115	1.4327
15	2.5	1.1269	1.1544	1.1895	1.2408	1.3334
20	2.5	1.1049	1.1280	1.1572	1.1999	1.2798
25	2.5	1.0920	1.1122	1.1374	1.1739	1.2430
30	2.5	1.0825	1.1012	1.1243	1.1581	1.2211
35	2.5	1.0758	1.0925	1.1137	1.1441	1.2026

Table A.6 Critical values for Z^* test statistic: Sample size N, shape parameter 3.0

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	3.0	1.2835	1.3449	1.4206	1.5298	1.7128
10	3.0	1.1650	1.2004	1.2450	1.3110	1.4304
15	3.0	1.1255	1.1529	1.1869	1.2366	1.3288
20	3.0	1.1045	1.1272	1.1561	1.1986	1.2761
25	3.0	1.0912	1.1117	1.1367	1.1736	1.2434
30	3.0	1.0822	1.1006	1.1230	1.1555	1.2183
35	3.0	1.0756	1.0919	1.1124	1.1439	1.1991

Table A.7 Critical values for Z^* test statistic: Sample size N, shape parameter 3.5

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	3.5	1.2820	1.3424	1.4184	1.5295	1.7164
10	3.5	1.1630	1.1989	1.2433	1.3065	1.4261
15	3.5	1.1248	1.1517	1.1852	1.2357	1.3272
20	3.5	1.1041	1.1271	1.1562	1.1972	1.2760
25	3.5	1.0910	1.1112	1.1356	1.1726	1.2400
30	3.5	1.0821	1.1004	1.1230	1.1559	1.2173
35	3.5	1.0752	1.0918	1.1121	1.1423	1.1984

Table A.8 Critical values for Z^* test statistic: Sample size N, shape parameter 4.0

Sample size	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4.0	1.2802	1.3411	1.4140	1.5244	1.7155
10	4.0	1.1631	1.1990	1.2426	1.3070	1.4244
15	4.0	1.1244	1.1509	1.1856	1.2346	1.3265
20	4.0	1.1036	1.1266	1.1552	1.1959	1.2743
25	4.0	1.0904	1.1109	1.1364	1.1730	1.2399
30	4.0	1.0814	1.0992	1.1218	1.1550	1.2156
35	4.0	1.0748	1.0910	1.1118	1.1423	1.1983

Appendix B. **Critical Value Tables for the Z* Test Statistic from Censored Samples**

Table B.1 Critical values for Z^* test statistic: Sample size N, shape parameter 0.5

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	0.5	1.4812	1.5582	1.6475	1.7593	1.8976
10	8	0.5	1.3697	1.4148	1.4690	1.5482	1.6834
15	12	0.5	1.3359	1.3726	1.4192	1.4844	1.5990
20	16	0.5	1.3168	1.3496	1.3916	1.4500	1.5546
25	20	0.5	1.3058	1.3356	1.3721	1.4246	1.5229
30	24	0.5	1.2970	1.3245	1.3589	1.4099	1.5001
35	28	0.5	1.2914	1.3167	1.3491	1.3956	1.4853

Table B.2 Critical values for Z^* test statistic: Sample size N, shape parameter 1.0

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	1.0	1.4831	1.5514	1.6328	1.7387	1.8832
10	8	1.0	1.3662	1.4046	1.4511	1.5182	1.6380
15	12	1.0	1.3260	1.3551	1.3909	1.4453	1.5402
20	16	1.0	1.3058	1.3308	1.3617	1.4050	1.4883
25	20	1.0	1.2925	1.3147	1.3412	1.3800	1.4547
30	24	1.0	1.2835	1.3036	1.3276	1.3627	1.4293
35	28	1.0	1.2764	1.2948	1.3178	1.3501	1.4125

Table B.3 Critical values for Z^* test statistic: Sample size N, shape parameter 1.5

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	1.5	1.4818	1.5499	1.6308	1.7344	1.8795
10	8	1.5	1.3625	1.3993	1.4445	1.5090	1.6252
15	12	1.5	1.3223	1.3493	1.3832	1.4325	1.5239
20	16	1.5	1.3008	1.3237	1.3531	1.3952	1.4722
25	20	1.5	1.2882	1.3080	1.3332	1.3701	1.4381
30	24	1.5	1.2791	1.2972	1.3194	1.3524	1.4147
35	28	1.5	1.2723	1.2886	1.3095	1.3389	1.3966

Table B.4 Critical values for Z^* test statistic: Sample size N, shape parameter 2.0

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	2.0	1.4806	1.5482	1.6271	1.7316	1.8789
10	8	2.0	1.3625	1.3983	1.4428	1.5041	1.6142
15	12	2.0	1.3213	1.3483	1.3812	1.4302	1.5181
20	16	2.0	1.2990	1.3212	1.3495	1.3898	1.4656
25	20	2.0	1.2869	1.3062	1.3307	1.3656	1.4323
30	24	2.0	1.2775	1.2956	1.3172	1.3486	1.4101
35	28	2.0	1.2703	1.2864	1.3061	1.3350	1.3900

Table B.5 Critical values for Z^* test statistic: Sample size N, shape parameter 2.5

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	2.5	1.4813	1.5477	1.6257	1.7323	1.8745
10	8	2.5	1.3606	1.3956	1.4388	1.4997	1.6134
15	12	2.5	1.3205	1.3469	1.3805	1.4282	1.5155
20	16	2.5	1.2988	1.3206	1.3482	1.3891	1.4635
25	20	2.5	1.2851	1.3046	1.3287	1.3633	1.4291
30	24	2.5	1.2767	1.2943	1.3161	1.3480	1.4064
35	28	2.5	1.2701	1.2861	1.3054	1.3347	1.3890

Table B.6 Critical values for Z^* test statistic: Sample size N, shape parameter 3.0

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	3.0	1.4805	1.5463	1.6256	1.7314	1.8758
10	8	3.0	1.3595	1.3938	1.4382	1.5001	1.6100
15	12	3.0	1.3199	1.3464	1.3781	1.4256	1.5104
20	16	3.0	1.2985	1.3203	1.3480	1.3886	1.4585
25	20	3.0	1.2857	1.3049	1.3290	1.3635	1.4279
30	24	3.0	1.2769	1.2934	1.3145	1.3461	1.4038
35	28	3.0	1.2698	1.2853	1.3046	1.3328	1.3866

Table B.7 Critical values for Z^* test statistic: Sample size N, shape parameter 3.5

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	3.5	1.4803	1.5459	1.6225	1.7285	1.8768
10	8	3.5	1.3585	1.3931	1.4365	1.4992	1.6091
15	12	3.5	1.3191	1.3454	1.3780	1.4259	1.5116
20	16	3.5	1.2985	1.3205	1.3477	1.3876	1.4611
25	20	3.5	1.2855	1.3044	1.3282	1.3629	1.4282
30	24	3.5	1.2759	1.2930	1.3139	1.3448	1.4038
35	28	3.5	1.2690	1.2846	1.3044	1.3331	1.3859

Table B.8 Critical values for Z^* test statistic: Sample size N, shape parameter 4.0

Size	Cnsr	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4	4.0	1.4780	1.5426	1.6226	1.7297	1.8770
10	8	4.0	1.3582	1.3938	1.4371	1.4996	1.6113
15	12	4.0	1.3190	1.3449	1.3779	1.4256	1.5138
20	16	4.0	1.2980	1.3202	1.3473	1.3874	1.4603
25	20	4.0	1.2851	1.3041	1.3274	1.3622	1.4256
30	24	4.0	1.2761	1.2933	1.3141	1.3451	1.4025
35	28	4.0	1.2694	1.2847	1.3042	1.3329	1.3856

Appendix C. Power Study of Z* Test Statistic for Complete Samples

Table C.1 Power Study: Sample size 5, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.149	0.099	0.050	0.010
Gamma(1.5)	0.441	0.353	0.256	0.142	0.031
Gamma(2.5)	0.523	0.431	0.325	0.186	0.044
Gamma(4.0)	0.579	0.489	0.375	0.224	0.054
Weibull(2.0)	0.619	0.527	0.413	0.252	0.062
Weibull(3.0)	0.699	0.616	0.506	0.337	0.099
Lognormal(0,2)	0.106	0.076	0.049	0.023	0.004
Lognormal(1,1)	0.306	0.237	0.164	0.082	0.016
Beta(2,2)	0.703	0.617	0.504	0.337	0.100
Beta(2,3)	0.651	0.561	0.448	0.281	0.076

Table C.2 Power Study: Sample size 10, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.098	0.049	0.009
Gamma(1.5)	0.767	0.689	0.577	0.408	0.142
Gamma(2.5)	0.871	0.816	0.729	0.572	0.255
Gamma(4.0)	0.915	0.875	0.806	0.671	0.353
Weibull(2.0)	0.941	0.909	0.849	0.727	0.407
Weibull(3.0)	0.973	0.954	0.920	0.840	0.573
Lognormal(0,2)	0.058	0.041	0.026	0.011	0.002
Lognormal(1,1)	0.486	0.404	0.308	0.191	0.051
Beta(2,2)	0.974	0.955	0.917	0.825	0.535
Beta(2,3)	0.955	0.927	0.876	0.761	0.439

Table C.3 Power Study: Sample size 15, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.149	0.100	0.051	0.010
Gamma(1.5)	0.921	0.879	0.808	0.670	0.352
Gamma(2.5)	0.975	0.957	0.924	0.846	0.583
Gamma(4.0)	0.988	0.978	0.958	0.905	0.703
Weibull(2.0)	0.994	0.988	0.974	0.937	0.762
Weibull(3.0)	0.998	0.996	0.992	0.976	0.884
Lognormal(0,2)	0.035	0.023	0.014	0.006	0.001
Lognormal(1,1)	0.639	0.562	0.466	0.327	0.119
Beta(2,2)	0.998	0.996	0.992	0.973	0.863
Beta(2,3)	0.996	0.992	0.981	0.951	0.793

Table C.4 Power Study: Sample size 20, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.149	0.099	0.049	0.010
Gamma(1.5)	0.976	0.960	0.927	0.844	0.580
Gamma(2.5)	0.996	0.992	0.984	0.955	0.816
Gamma(4.0)	0.998	0.997	0.993	0.980	0.900
Weibull(2.0)	0.999	0.998	0.996	0.988	0.928
Weibull(3.0)	1.000	1.000	0.999	0.997	0.978
Lognormal(0,2)	0.025	0.017	0.010	0.004	0.001
Lognormal(1,1)	0.759	0.697	0.610	0.463	0.216
Beta(2,2)	1.000	1.000	0.999	0.997	0.971
Beta(2,3)	1.000	0.999	0.998	0.992	0.944

Table C.5 Power Study: Sample size 25, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.153	0.101	0.049	0.010
Gamma(1.5)	0.994	0.989	0.976	0.937	0.751
Gamma(2.5)	0.999	0.999	0.997	0.989	0.928
Gamma(4.0)	1.000	1.000	0.999	0.996	0.968
Weibull(2.0)	1.000	1.000	1.000	0.998	0.982
Weibull(3.0)	1.000	1.000	1.000	1.000	0.996
Lognormal(0,2)	0.018	0.013	0.008	0.003	0.000
Lognormal(1,1)	0.846	0.801	0.729	0.599	0.322
Beta(2,2)	1.000	1.000	1.000	1.000	0.995
Beta(2,3)	1.000	1.000	1.000	0.999	0.986

Table C.6 Power Study: Sample size 30, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.148	0.098	0.050	0.010
Gamma(1.5)	0.998	0.997	0.993	0.978	0.876
Gamma(2.5)	1.000	1.000	0.999	0.997	0.977
Gamma(4.0)	1.000	1.000	1.000	0.999	0.993
Weibull(2.0)	1.000	1.000	1.000	1.000	0.996
Weibull(3.0)	1.000	1.000	1.000	1.000	1.000
Lognormal(0,2)	0.015	0.010	0.006	0.003	0.000
Lognormal(1,1)	0.904	0.869	0.815	0.709	0.453
Beta(2,2)	1.000	1.000	1.000	1.000	0.999
Beta(2,3)	1.000	1.000	1.000	1.000	0.998

Table C.7 Power Study: Sample size 35, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.153	0.103	0.051	0.011
Gamma(1.5)	1.000	0.999	0.998	0.992	0.943
Gamma(2.5)	1.000	1.000	1.000	0.999	0.993
Gamma(4.0)	1.000	1.000	1.000	1.000	0.998
Weibull(2.0)	1.000	1.000	1.000	1.000	0.999
Weibull(3.0)	1.000	1.000	1.000	1.000	1.000
Lognormal(0,2)	0.011	0.007	0.005	0.002	0.000
Lognormal(1,1)	0.941	0.918	0.877	0.792	0.562
Beta(2,2)	1.000	1.000	1.000	1.000	1.000
Beta(2,3)	1.000	1.000	1.000	1.000	1.000

Table C.8 Power Study: Sample size 5, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.149	0.098	0.050	0.009
Gamma(1.5)	0.259	0.199	0.135	0.069	0.014
Gamma(2.5)	0.325	0.254	0.178	0.096	0.020
Gamma(4.0)	0.377	0.302	0.216	0.118	0.025
Weibull(2.0)	0.419	0.340	0.244	0.136	0.030
Weibull(3.0)	0.516	0.434	0.330	0.202	0.048
Lognormal(0,2)	0.052	0.036	0.023	0.010	0.002
Lognormal(1,1)	0.164	0.123	0.079	0.039	0.007
Beta(2,2)	0.525	0.440	0.337	0.205	0.051
Beta(2,3)	0.460	0.377	0.278	0.157	0.037

Table C.9 Power Study: Sample size 10, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.195	0.145	0.097	0.048	0.010
Gamma(1.5)	0.338	0.268	0.190	0.104	0.025
Gamma(2.5)	0.505	0.423	0.323	0.198	0.057
Gamma(4.0)	0.617	0.537	0.435	0.292	0.099
Weibull(2.0)	0.699	0.623	0.520	0.362	0.134
Weibull(3.0)	0.840	0.784	0.702	0.556	0.278
Lognormal(0,2)	0.009	0.006	0.003	0.001	0.000
Lognormal(1,1)	0.136	0.102	0.066	0.032	0.007
Beta(2,2)	0.844	0.786	0.698	0.544	0.253
Beta(2,3)	0.761	0.688	0.585	0.421	0.163

Table C.10 Power Study: Sample size 15, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.151	0.101	0.050	0.010
Gamma(1.5)	0.414	0.337	0.250	0.146	0.039
Gamma(2.5)	0.661	0.582	0.479	0.331	0.122
Gamma(4.0)	0.784	0.722	0.634	0.484	0.228
Weibull(2.0)	0.864	0.813	0.733	0.591	0.302
Weibull(3.0)	0.955	0.934	0.893	0.810	0.571
Lognormal(0,2)	0.002	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.124	0.091	0.060	0.031	0.006
Beta(2,2)	0.959	0.937	0.896	0.802	0.536
Beta(2,3)	0.910	0.869	0.803	0.672	0.373

Table C.11 Power Study: Sample size 20, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.148	0.097	0.049	0.009
Gamma(1.5)	0.478	0.399	0.303	0.184	0.056
Gamma(2.5)	0.764	0.697	0.601	0.451	0.201
Gamma(4.0)	0.885	0.841	0.774	0.652	0.379
Weibull(2.0)	0.939	0.909	0.860	0.757	0.485
Weibull(3.0)	0.988	0.980	0.966	0.926	0.778
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.109	0.081	0.053	0.026	0.005
Beta(2,2)	0.990	0.983	0.966	0.921	0.750
Beta(2,3)	0.967	0.948	0.912	0.829	0.576

Table C.12 Power Study: Sample size 25, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.149	0.099	0.049	0.010
Gamma(1.5)	0.541	0.457	0.358	0.230	0.074
Gamma(2.5)	0.844	0.788	0.707	0.567	0.292
Gamma(4.0)	0.939	0.912	0.866	0.769	0.521
Weibull(2.0)	0.976	0.961	0.933	0.864	0.647
Weibull(3.0)	0.997	0.995	0.990	0.974	0.901
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.101	0.074	0.049	0.024	0.005
Beta(2,2)	0.998	0.996	0.990	0.974	0.883
Beta(2,3)	0.989	0.981	0.964	0.920	0.744

Table C.13 Power Study: Sample size 30, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.153	0.101	0.051	0.010
Gamma(1.5)	0.594	0.515	0.414	0.277	0.097
Gamma(2.5)	0.899	0.857	0.794	0.677	0.398
Gamma(4.0)	0.970	0.953	0.923	0.859	0.654
Weibull(2.0)	0.990	0.983	0.969	0.933	0.776
Weibull(3.0)	1.000	0.999	0.997	0.992	0.960
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.094	0.069	0.045	0.023	0.005
Beta(2,2)	1.000	0.999	0.998	0.992	0.953
Beta(2,3)	0.997	0.994	0.987	0.967	0.858

Table C.14 Power Study: Sample size 35, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.151	0.101	0.050	0.009
Gamma(1.5)	0.639	0.564	0.462	0.311	0.110
Gamma(2.5)	0.933	0.904	0.853	0.747	0.476
Gamma(4.0)	0.984	0.974	0.957	0.912	0.748
Weibull(2.0)	0.996	0.992	0.985	0.964	0.861
Weibull(3.0)	1.000	1.000	0.999	0.997	0.984
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.087	0.064	0.042	0.020	0.004
Beta(2,2)	1.000	1.000	0.999	0.997	0.979
Beta(2,3)	0.999	0.998	0.996	0.986	0.920

Table C.15 Power Study: Sample size 5, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.100	0.050	0.009
Gamma(1.5)	0.202	0.153	0.103	0.051	0.010
Gamma(2.5)	0.257	0.197	0.135	0.068	0.012
Gamma(4.0)	0.300	0.237	0.166	0.086	0.017
Weibull(2.0)	0.340	0.270	0.193	0.102	0.021
Weibull(3.0)	0.436	0.360	0.269	0.153	0.034
Lognormal(0,2)	0.038	0.027	0.017	0.008	0.001
Lognormal(1,1)	0.123	0.091	0.059	0.029	0.006
Beta(2,2)	0.446	0.367	0.273	0.159	0.038
Beta(2,3)	0.383	0.308	0.223	0.122	0.025

Table C.16 Power Study: Sample size 10, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.147	0.098	0.048	0.009
Gamma(1.5)	0.199	0.150	0.099	0.050	0.010
Gamma(2.5)	0.331	0.262	0.185	0.100	0.023
Gamma(4.0)	0.444	0.366	0.273	0.165	0.045
Weibull(2.0)	0.538	0.453	0.349	0.214	0.061
Weibull(3.0)	0.725	0.651	0.550	0.397	0.161
Lognormal(0,2)	0.004	0.003	0.001	0.000	0.000
Lognormal(1,1)	0.070	0.050	0.030	0.014	0.002
Beta(2,2)	0.734	0.660	0.556	0.394	0.150
Beta(2,3)	0.615	0.530	0.419	0.267	0.084

Table C.17 Power Study: Sample size 15, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.152	0.101	0.050	0.011
Gamma(1.5)	0.200	0.151	0.101	0.050	0.011
Gamma(2.5)	0.401	0.326	0.243	0.140	0.039
Gamma(4.0)	0.559	0.484	0.384	0.247	0.086
Weibull(2.0)	0.681	0.603	0.499	0.345	0.132
Weibull(3.0)	0.874	0.830	0.761	0.632	0.362
Lognormal(0,2)	0.001	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.043	0.030	0.018	0.008	0.001
Beta(2,2)	0.886	0.839	0.764	0.622	0.331
Beta(2,3)	0.774	0.704	0.603	0.440	0.183

Table C.18 Power Study: Sample size 20, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.151	0.101	0.051	0.009
Gamma(1.5)	0.205	0.153	0.102	0.052	0.009
Gamma(2.5)	0.466	0.388	0.298	0.184	0.051
Gamma(4.0)	0.657	0.582	0.486	0.345	0.129
Weibull(2.0)	0.780	0.715	0.621	0.467	0.196
Weibull(3.0)	0.946	0.921	0.878	0.788	0.531
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.029	0.020	0.012	0.005	0.001
Beta(2,2)	0.952	0.928	0.884	0.788	0.500
Beta(2,3)	0.873	0.824	0.745	0.596	0.284

Table C.19 Power Study: Sample size 25, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.154	0.105	0.052	0.010
Gamma(1.5)	0.200	0.151	0.101	0.051	0.010
Gamma(2.5)	0.510	0.431	0.340	0.216	0.068
Gamma(4.0)	0.732	0.668	0.574	0.427	0.186
Weibull(2.0)	0.855	0.803	0.724	0.580	0.292
Weibull(3.0)	0.977	0.965	0.942	0.887	0.701
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.019	0.013	0.007	0.003	0.001
Beta(2,2)	0.981	0.968	0.944	0.884	0.665
Beta(2,3)	0.929	0.894	0.835	0.713	0.415

Table C.20 Power Study: Sample size 30, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.206	0.154	0.103	0.050	0.009
Gamma(1.5)	0.201	0.151	0.102	0.051	0.010
Gamma(2.5)	0.559	0.482	0.385	0.253	0.084
Gamma(4.0)	0.788	0.729	0.644	0.503	0.243
Weibull(2.0)	0.904	0.864	0.800	0.674	0.380
Weibull(3.0)	0.990	0.984	0.972	0.940	0.807
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.012	0.008	0.004	0.002	0.000
Beta(2,2)	0.992	0.986	0.975	0.940	0.786
Beta(2,3)	0.959	0.936	0.896	0.802	0.524

Table C.21 Power Study: Sample size 35, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.149	0.100	0.050	0.010
Gamma(1.5)	0.198	0.147	0.098	0.049	0.010
Gamma(2.5)	0.604	0.526	0.424	0.285	0.103
Gamma(4.0)	0.840	0.785	0.707	0.572	0.309
Weibull(2.0)	0.936	0.905	0.854	0.748	0.477
Weibull(3.0)	0.996	0.993	0.987	0.969	0.886
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.010	0.006	0.004	0.001	0.000
Beta(2,2)	0.997	0.995	0.989	0.970	0.872
Beta(2,3)	0.979	0.965	0.937	0.870	0.639

Table C.22 Power Study: Sample size 5, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.048	0.010
Gamma(1.5)	0.170	0.127	0.085	0.042	0.008
Gamma(2.5)	0.220	0.164	0.112	0.057	0.011
Gamma(4.0)	0.260	0.201	0.138	0.070	0.016
Weibull(2.0)	0.297	0.232	0.162	0.083	0.018
Weibull(3.0)	0.391	0.315	0.229	0.126	0.029
Lognormal(0,2)	0.031	0.022	0.013	0.005	0.001
Lognormal(1,1)	0.102	0.073	0.047	0.022	0.004
Beta(2,2)	0.405	0.326	0.238	0.133	0.033
Beta(2,3)	0.335	0.263	0.186	0.099	0.022

Table C.23 Power Study: Sample size 10, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.195	0.145	0.096	0.048	0.009
Gamma(1.5)	0.140	0.102	0.065	0.032	0.006
Gamma(2.5)	0.245	0.188	0.127	0.065	0.015
Gamma(4.0)	0.343	0.274	0.195	0.108	0.027
Weibull(2.0)	0.430	0.349	0.258	0.150	0.040
Weibull(3.0)	0.632	0.555	0.451	0.305	0.109
Lognormal(0,2)	0.003	0.002	0.001	0.000	0.000
Lognormal(1,1)	0.046	0.032	0.019	0.008	0.001
Beta(2,2)	0.653	0.571	0.460	0.308	0.105
Beta(2,3)	0.517	0.430	0.326	0.197	0.056

Table C.24 Power Study: Sample size 15, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.149	0.099	0.050	0.010
Gamma(1.5)	0.119	0.085	0.054	0.025	0.004
Gamma(2.5)	0.270	0.207	0.144	0.077	0.017
Gamma(4.0)	0.412	0.337	0.252	0.150	0.041
Weibull(2.0)	0.537	0.454	0.353	0.223	0.067
Weibull(3.0)	0.792	0.731	0.645	0.500	0.236
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.023	0.016	0.009	0.003	0.001
Beta(2,2)	0.807	0.743	0.649	0.494	0.212
Beta(2,3)	0.653	0.569	0.461	0.307	0.101

Table C.25 Power Study: Sample size 20, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.148	0.101	0.051	0.010
Gamma(1.5)	0.104	0.073	0.046	0.021	0.004
Gamma(2.5)	0.294	0.227	0.162	0.089	0.020
Gamma(4.0)	0.476	0.399	0.310	0.192	0.058
Weibull(2.0)	0.624	0.543	0.439	0.293	0.098
Weibull(3.0)	0.884	0.842	0.776	0.652	0.373
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.012	0.007	0.004	0.001	0.000
Beta(2,2)	0.898	0.855	0.786	0.650	0.346
Beta(2,3)	0.759	0.684	0.583	0.422	0.163

Table C.26 Power Study: Sample size 25, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.204	0.153	0.102	0.051	0.011
Gamma(1.5)	0.096	0.066	0.040	0.017	0.003
Gamma(2.5)	0.309	0.241	0.169	0.096	0.023
Gamma(4.0)	0.533	0.453	0.357	0.231	0.077
Weibull(2.0)	0.697	0.621	0.514	0.365	0.141
Weibull(3.0)	0.938	0.909	0.861	0.767	0.517
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.007	0.004	0.002	0.001	0.000
Beta(2,2)	0.947	0.917	0.867	0.763	0.481
Beta(2,3)	0.832	0.767	0.675	0.519	0.232

Table C.27 Power Study: Sample size 30, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.153	0.103	0.051	0.010
Gamma(1.5)	0.084	0.058	0.035	0.015	0.002
Gamma(2.5)	0.324	0.258	0.186	0.104	0.025
Gamma(4.0)	0.573	0.498	0.403	0.270	0.094
Weibull(2.0)	0.758	0.688	0.589	0.435	0.180
Weibull(3.0)	0.966	0.948	0.916	0.846	0.628
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.004	0.003	0.002	0.000	0.000
Beta(2,2)	0.974	0.957	0.925	0.848	0.597
Beta(2,3)	0.881	0.830	0.751	0.603	0.295

Table C.28 Power Study: Sample size 35, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.150	0.100	0.051	0.010
Gamma(1.5)	0.076	0.052	0.031	0.013	0.002
Gamma(2.5)	0.337	0.268	0.194	0.110	0.027
Gamma(4.0)	0.621	0.545	0.451	0.313	0.118
Weibull(2.0)	0.806	0.742	0.650	0.501	0.227
Weibull(3.0)	0.981	0.970	0.950	0.902	0.730
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.002	0.001	0.000	0.000	0.000
Beta(2,2)	0.986	0.977	0.957	0.907	0.707
Beta(2,3)	0.921	0.882	0.816	0.690	0.383

Table C.29 Power Study: Sample size 5, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.149	0.099	0.051	0.011
Gamma(1.5)	0.155	0.114	0.075	0.037	0.007
Gamma(2.5)	0.198	0.149	0.100	0.051	0.010
Gamma(4.0)	0.234	0.177	0.122	0.063	0.013
Weibull(2.0)	0.269	0.208	0.142	0.074	0.016
Weibull(3.0)	0.363	0.291	0.208	0.115	0.026
Lognormal(0,2)	0.028	0.020	0.012	0.005	0.001
Lognormal(1,1)	0.092	0.066	0.042	0.020	0.004
Beta(2,2)	0.375	0.301	0.215	0.120	0.030
Beta(2,3)	0.309	0.242	0.168	0.089	0.020

Table C.30 Power Study: Sample size 10, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.101	0.049	0.009
Gamma(1.5)	0.108	0.077	0.049	0.022	0.004
Gamma(2.5)	0.193	0.144	0.096	0.047	0.009
Gamma(4.0)	0.286	0.222	0.156	0.085	0.019
Weibull(2.0)	0.363	0.291	0.209	0.117	0.028
Weibull(3.0)	0.569	0.490	0.391	0.255	0.082
Lognormal(0,2)	0.002	0.001	0.001	0.000	0.000
Lognormal(1,1)	0.036	0.023	0.014	0.006	0.001
Beta(2,2)	0.591	0.507	0.399	0.254	0.079
Beta(2,3)	0.448	0.364	0.268	0.157	0.040

Table C.31 Power Study: Sample size 15, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.194	0.145	0.095	0.047	0.009
Gamma(1.5)	0.083	0.057	0.034	0.015	0.002
Gamma(2.5)	0.196	0.146	0.097	0.047	0.010
Gamma(4.0)	0.321	0.253	0.179	0.098	0.024
Weibull(2.0)	0.439	0.359	0.265	0.153	0.040
Weibull(3.0)	0.720	0.649	0.549	0.399	0.168
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.015	0.009	0.005	0.002	0.000
Beta(2,2)	0.740	0.666	0.562	0.398	0.152
Beta(2,3)	0.560	0.473	0.362	0.223	0.064

Table C.32 Power Study: Sample size 20, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.100	0.050	0.010
Gamma(1.5)	0.065	0.045	0.027	0.011	0.001
Gamma(2.5)	0.202	0.153	0.103	0.052	0.010
Gamma(4.0)	0.364	0.293	0.212	0.121	0.032
Weibull(2.0)	0.507	0.425	0.325	0.201	0.057
Weibull(3.0)	0.821	0.767	0.681	0.538	0.266
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.007	0.004	0.002	0.001	0.000
Beta(2,2)	0.844	0.786	0.696	0.540	0.245
Beta(2,3)	0.660	0.577	0.466	0.309	0.100

Table C.33 Power Study: Sample size 25, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.153	0.101	0.051	0.011
Gamma(1.5)	0.052	0.035	0.020	0.008	0.001
Gamma(2.5)	0.198	0.148	0.099	0.050	0.010
Gamma(4.0)	0.392	0.319	0.236	0.139	0.038
Weibull(2.0)	0.566	0.480	0.379	0.246	0.079
Weibull(3.0)	0.886	0.845	0.777	0.660	0.386
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.003	0.002	0.001	0.000	0.000
Beta(2,2)	0.904	0.862	0.791	0.657	0.361
Beta(2,3)	0.728	0.653	0.545	0.385	0.142

Table C.34 Power Study: Sample size 30, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.152	0.100	0.049	0.010
Gamma(1.5)	0.043	0.028	0.017	0.007	0.001
Gamma(2.5)	0.201	0.150	0.100	0.051	0.010
Gamma(4.0)	0.420	0.345	0.260	0.156	0.044
Weibull(2.0)	0.619	0.534	0.429	0.283	0.093
Weibull(3.0)	0.926	0.895	0.843	0.739	0.475
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.002	0.001	0.000	0.000	0.000
Beta(2,2)	0.943	0.913	0.861	0.752	0.460
Beta(2,3)	0.785	0.712	0.613	0.449	0.178

Table C.35 Power Study: Sample size 35, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.151	0.101	0.050	0.010
Gamma(1.5)	0.034	0.023	0.012	0.005	0.001
Gamma(2.5)	0.199	0.149	0.100	0.050	0.010
Gamma(4.0)	0.450	0.373	0.282	0.176	0.052
Weibull(2.0)	0.661	0.581	0.474	0.328	0.116
Weibull(3.0)	0.954	0.932	0.892	0.813	0.578
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.001	0.001	0.000	0.000	0.000
Beta(2,2)	0.966	0.945	0.909	0.823	0.552
Beta(2,3)	0.837	0.776	0.682	0.525	0.231

Table C.36 Power Study: Sample size 5, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.149	0.099	0.049	0.010
Gamma(1.5)	0.142	0.103	0.067	0.033	0.006
Gamma(2.5)	0.182	0.136	0.089	0.044	0.008
Gamma(4.0)	0.214	0.161	0.111	0.057	0.012
Weibull(2.0)	0.250	0.191	0.130	0.066	0.013
Weibull(3.0)	0.341	0.269	0.192	0.104	0.022
Lognormal(0,2)	0.026	0.017	0.010	0.004	0.001
Lognormal(1,1)	0.082	0.059	0.037	0.017	0.003
Beta(2,2)	0.354	0.279	0.199	0.110	0.026
Beta(2,3)	0.289	0.224	0.156	0.080	0.016

Table C.37 Power Study: Sample size 10, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.149	0.098	0.049	0.009
Gamma(1.5)	0.091	0.065	0.040	0.017	0.003
Gamma(2.5)	0.168	0.124	0.081	0.037	0.007
Gamma(4.0)	0.244	0.186	0.127	0.066	0.013
Weibull(2.0)	0.319	0.251	0.177	0.096	0.021
Weibull(3.0)	0.523	0.442	0.346	0.218	0.063
Lognormal(0,2)	0.001	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.028	0.019	0.011	0.005	0.001
Beta(2,2)	0.545	0.460	0.357	0.220	0.062
Beta(2,3)	0.401	0.319	0.231	0.128	0.030

Table C.38 Power Study: Sample size 15, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.050	0.011
Gamma(1.5)	0.062	0.042	0.025	0.011	0.002
Gamma(2.5)	0.158	0.116	0.075	0.036	0.007
Gamma(4.0)	0.262	0.201	0.140	0.075	0.017
Weibull(2.0)	0.375	0.300	0.216	0.122	0.029
Weibull(3.0)	0.665	0.589	0.491	0.346	0.133
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.011	0.007	0.003	0.001	0.000
Beta(2,2)	0.688	0.608	0.501	0.346	0.119
Beta(2,3)	0.493	0.407	0.305	0.181	0.047

Table C.39 Power Study: Sample size 20, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.100	0.049	0.009
Gamma(1.5)	0.045	0.030	0.017	0.007	0.001
Gamma(2.5)	0.152	0.112	0.072	0.034	0.005
Gamma(4.0)	0.286	0.224	0.158	0.085	0.021
Weibull(2.0)	0.421	0.343	0.250	0.144	0.037
Weibull(3.0)	0.761	0.697	0.602	0.452	0.203
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.004	0.003	0.001	0.000	0.000
Beta(2,2)	0.789	0.722	0.622	0.458	0.189
Beta(2,3)	0.581	0.495	0.385	0.240	0.071

Table C.40 Power Study: Sample size 25, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.151	0.101	0.050	0.009
Gamma(1.5)	0.034	0.022	0.012	0.005	0.001
Gamma(2.5)	0.140	0.101	0.064	0.030	0.005
Gamma(4.0)	0.301	0.235	0.167	0.091	0.021
Weibull(2.0)	0.464	0.382	0.287	0.172	0.047
Weibull(3.0)	0.839	0.783	0.703	0.561	0.282
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.002	0.001	0.001	0.000	0.000
Beta(2,2)	0.860	0.804	0.718	0.567	0.268
Beta(2,3)	0.644	0.558	0.447	0.293	0.089

Table C.41 Power Study: Sample size 30, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.148	0.099	0.052	0.010
Gamma(1.5)	0.026	0.017	0.010	0.004	0.001
Gamma(2.5)	0.136	0.098	0.064	0.030	0.005
Gamma(4.0)	0.318	0.250	0.180	0.102	0.025
Weibull(2.0)	0.506	0.422	0.323	0.198	0.056
Weibull(3.0)	0.883	0.839	0.771	0.647	0.371
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.001	0.000	0.000	0.000	0.000
Beta(2,2)	0.908	0.864	0.795	0.666	0.357
Beta(2,3)	0.699	0.617	0.507	0.348	0.117

Table C.42 Power Study: Sample size 35, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.196	0.148	0.100	0.050	0.010
Gamma(1.5)	0.020	0.012	0.006	0.002	0.000
Gamma(2.5)	0.127	0.092	0.058	0.026	0.005
Gamma(4.0)	0.327	0.262	0.189	0.105	0.028
Weibull(2.0)	0.535	0.456	0.357	0.219	0.069
Weibull(3.0)	0.921	0.888	0.834	0.722	0.460
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.000	0.000	0.000	0.000	0.000
Beta(2,2)	0.938	0.907	0.853	0.733	0.448
Beta(2,3)	0.748	0.672	0.567	0.397	0.156

Table C.43 Power Study: Sample size 5, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.151	0.100	0.049	0.010
Gamma(1.5)	0.134	0.097	0.063	0.030	0.005
Gamma(2.5)	0.173	0.127	0.084	0.041	0.008
Gamma(4.0)	0.208	0.157	0.104	0.051	0.010
Weibull(2.0)	0.237	0.181	0.122	0.062	0.011
Weibull(3.0)	0.325	0.255	0.181	0.097	0.020
Lognormal(0,2)	0.024	0.016	0.010	0.004	0.001
Lognormal(1,1)	0.081	0.057	0.035	0.016	0.002
Beta(2,2)	0.339	0.269	0.190	0.103	0.023
Beta(2,3)	0.272	0.209	0.143	0.073	0.014

Table C.44 Power Study: Sample size 10, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.151	0.101	0.052	0.010
Gamma(1.5)	0.079	0.055	0.033	0.014	0.002
Gamma(2.5)	0.148	0.108	0.069	0.032	0.006
Gamma(4.0)	0.218	0.165	0.112	0.058	0.012
Weibull(2.0)	0.289	0.222	0.154	0.083	0.017
Weibull(3.0)	0.488	0.409	0.312	0.196	0.057
Lognormal(0,2)	0.001	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.024	0.016	0.009	0.004	0.001
Beta(2,2)	0.515	0.431	0.327	0.201	0.054
Beta(2,3)	0.368	0.289	0.204	0.112	0.025

Table C.45 Power Study: Sample size 15, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.097	0.048	0.010
Gamma(1.5)	0.050	0.034	0.020	0.008	0.001
Gamma(2.5)	0.130	0.094	0.060	0.028	0.004
Gamma(4.0)	0.223	0.170	0.116	0.059	0.013
Weibull(2.0)	0.329	0.258	0.183	0.096	0.022
Weibull(3.0)	0.619	0.542	0.443	0.297	0.107
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.008	0.005	0.003	0.001	0.000
Beta(2,2)	0.643	0.560	0.454	0.300	0.095
Beta(2,3)	0.442	0.359	0.264	0.148	0.035

Table C.46 Power Study: Sample size 20, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.147	0.097	0.049	0.009
Gamma(1.5)	0.034	0.023	0.013	0.005	0.001
Gamma(2.5)	0.119	0.085	0.052	0.025	0.004
Gamma(4.0)	0.233	0.178	0.121	0.063	0.013
Weibull(2.0)	0.358	0.282	0.201	0.112	0.026
Weibull(3.0)	0.712	0.638	0.537	0.389	0.158
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.003	0.002	0.001	0.000	0.000
Beta(2,2)	0.745	0.669	0.557	0.396	0.147
Beta(2,3)	0.516	0.427	0.321	0.192	0.049

Table C.47 Power Study: Sample size 25, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.149	0.101	0.050	0.011
Gamma(1.5)	0.025	0.015	0.009	0.003	0.001
Gamma(2.5)	0.105	0.074	0.046	0.020	0.004
Gamma(4.0)	0.238	0.182	0.125	0.064	0.015
Weibull(2.0)	0.389	0.312	0.231	0.130	0.033
Weibull(3.0)	0.787	0.723	0.634	0.488	0.229
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.001	0.001	0.000	0.000	0.000
Beta(2,2)	0.818	0.753	0.660	0.498	0.219
Beta(2,3)	0.572	0.481	0.375	0.231	0.066

Table C.48 Power Study: Sample size 30, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.148	0.099	0.050	0.010
Gamma(1.5)	0.017	0.011	0.006	0.002	0.000
Gamma(2.5)	0.097	0.069	0.041	0.019	0.003
Gamma(4.0)	0.247	0.190	0.131	0.069	0.015
Weibull(2.0)	0.416	0.336	0.245	0.141	0.036
Weibull(3.0)	0.840	0.785	0.705	0.567	0.291
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.001	0.000	0.000	0.000	0.000
Beta(2,2)	0.871	0.817	0.735	0.587	0.286
Beta(2,3)	0.621	0.535	0.422	0.268	0.080

Table C.49 Power Study: Sample size 35, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.194	0.145	0.099	0.049	0.010
Gamma(1.5)	0.012	0.008	0.004	0.001	0.000
Gamma(2.5)	0.089	0.062	0.037	0.016	0.002
Gamma(4.0)	0.249	0.191	0.134	0.071	0.016
Weibull(2.0)	0.447	0.363	0.273	0.159	0.042
Weibull(3.0)	0.880	0.835	0.767	0.642	0.363
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.000	0.000	0.000	0.000	0.000
Beta(2,2)	0.908	0.867	0.798	0.663	0.361
Beta(2,3)	0.671	0.584	0.474	0.316	0.103

Table C.50 Power Study: Sample size 5, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.151	0.103	0.051	0.009
Gamma(1.5)	0.128	0.094	0.060	0.027	0.005
Gamma(2.5)	0.165	0.121	0.081	0.039	0.007
Gamma(4.0)	0.195	0.146	0.098	0.049	0.009
Weibull(2.0)	0.232	0.173	0.116	0.058	0.011
Weibull(3.0)	0.313	0.244	0.175	0.093	0.019
Lognormal(0,2)	0.022	0.015	0.009	0.004	0.001
Lognormal(1,1)	0.077	0.054	0.034	0.016	0.002
Beta(2,2)	0.328	0.256	0.183	0.097	0.021
Beta(2,3)	0.265	0.202	0.139	0.072	0.013

Table C.51 Power Study: Sample size 10, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.146	0.098	0.049	0.010
Gamma(1.5)	0.070	0.049	0.029	0.013	0.002
Gamma(2.5)	0.132	0.095	0.060	0.028	0.005
Gamma(4.0)	0.199	0.148	0.099	0.049	0.010
Weibull(2.0)	0.264	0.201	0.137	0.072	0.015
Weibull(3.0)	0.458	0.378	0.289	0.172	0.047
Lognormal(0,2)	0.001	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.021	0.014	0.008	0.003	0.000
Beta(2,2)	0.485	0.400	0.303	0.179	0.048
Beta(2,3)	0.339	0.262	0.183	0.097	0.021

Table C.52 Power Study: Sample size 15, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.194	0.146	0.097	0.049	0.010
Gamma(1.5)	0.042	0.028	0.016	0.007	0.001
Gamma(2.5)	0.110	0.080	0.049	0.022	0.004
Gamma(4.0)	0.194	0.148	0.098	0.049	0.009
Weibull(2.0)	0.292	0.228	0.154	0.081	0.017
Weibull(3.0)	0.581	0.502	0.400	0.263	0.089
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.006	0.004	0.002	0.001	0.000
Beta(2,2)	0.607	0.523	0.412	0.266	0.081
Beta(2,3)	0.401	0.320	0.228	0.127	0.028

Table C.53 Power Study: Sample size 20, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.049	0.010
Gamma(1.5)	0.027	0.017	0.009	0.003	0.000
Gamma(2.5)	0.098	0.069	0.041	0.018	0.003
Gamma(4.0)	0.200	0.148	0.099	0.051	0.009
Weibull(2.0)	0.313	0.245	0.170	0.091	0.019
Weibull(3.0)	0.667	0.588	0.488	0.344	0.129
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.002	0.001	0.001	0.000	0.000
Beta(2,2)	0.706	0.623	0.511	0.353	0.122
Beta(2,3)	0.466	0.379	0.279	0.161	0.038

Table C.54 Power Study: Sample size 25, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.148	0.098	0.048	0.010
Gamma(1.5)	0.018	0.011	0.006	0.002	0.000
Gamma(2.5)	0.082	0.055	0.033	0.014	0.002
Gamma(4.0)	0.196	0.145	0.096	0.047	0.010
Weibull(2.0)	0.336	0.262	0.181	0.098	0.023
Weibull(3.0)	0.742	0.674	0.575	0.423	0.180
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.001	0.001	0.000	0.000	0.000
Beta(2,2)	0.781	0.707	0.603	0.439	0.176
Beta(2,3)	0.516	0.425	0.316	0.186	0.048

Table C.55 Power Study: Sample size 30, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.148	0.098	0.048	0.010
Gamma(1.5)	0.013	0.008	0.004	0.001	0.000
Gamma(2.5)	0.075	0.051	0.031	0.013	0.002
Gamma(4.0)	0.201	0.150	0.100	0.050	0.010
Weibull(2.0)	0.356	0.282	0.199	0.110	0.026
Weibull(3.0)	0.798	0.738	0.650	0.500	0.238
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.000	0.000	0.000	0.000	0.000
Beta(2,2)	0.837	0.776	0.683	0.523	0.235
Beta(2,3)	0.561	0.474	0.363	0.221	0.061

Table C.56 Power Study: Sample size 35, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.149	0.098	0.049	0.010
Gamma(1.5)	0.008	0.005	0.003	0.001	0.000
Gamma(2.5)	0.065	0.044	0.025	0.010	0.002
Gamma(4.0)	0.200	0.152	0.101	0.051	0.010
Weibull(2.0)	0.376	0.302	0.215	0.118	0.028
Weibull(3.0)	0.841	0.790	0.710	0.567	0.292
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.000	0.000	0.000	0.000	0.000
Beta(2,2)	0.880	0.828	0.746	0.596	0.292
Beta(2,3)	0.606	0.517	0.403	0.251	0.073

Appendix D. **Power Study of Z* Test Statistic for Censored Samples**

Table D.1 Power Study: Sample size 5, observations 4, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.099	0.049	0.010
Gamma(1.5)	0.361	0.281	0.194	0.101	0.021
Gamma(2.5)	0.418	0.333	0.235	0.125	0.027
Gamma(4.0)	0.454	0.366	0.263	0.141	0.030
Weibull(2.0)	0.471	0.380	0.275	0.151	0.033
Weibull(3.0)	0.537	0.446	0.336	0.195	0.044
Lognormal(0,2)	0.155	0.114	0.074	0.036	0.007
Lognormal(1,1)	0.298	0.228	0.153	0.077	0.015
Beta(2,2)	0.521	0.430	0.319	0.181	0.042
Beta(2,3)	0.484	0.395	0.287	0.158	0.035

Table D.2 Power Study: Sample size 10, observations 8, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.151	0.102	0.049	0.010
Gamma(1.5)	0.658	0.575	0.465	0.303	0.091
Gamma(2.5)	0.765	0.694	0.594	0.423	0.154
Gamma(4.0)	0.824	0.761	0.670	0.509	0.216
Weibull(2.0)	0.848	0.789	0.701	0.539	0.231
Weibull(3.0)	0.908	0.865	0.798	0.662	0.353
Lognormal(0,2)	0.129	0.093	0.060	0.028	0.005
Lognormal(1,1)	0.507	0.423	0.326	0.196	0.051
Beta(2,2)	0.887	0.837	0.756	0.607	0.287
Beta(2,3)	0.852	0.794	0.705	0.542	0.230

Table D.3 Power Study: Sample size 15, observations 12, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.196	0.149	0.099	0.050	0.011
Gamma(1.5)	0.845	0.785	0.691	0.529	0.235
Gamma(2.5)	0.930	0.897	0.834	0.713	0.406
Gamma(4.0)	0.956	0.932	0.887	0.791	0.518
Weibull(2.0)	0.966	0.947	0.908	0.819	0.543
Weibull(3.0)	0.986	0.976	0.956	0.903	0.706
Lognormal(0,2)	0.117	0.085	0.053	0.026	0.005
Lognormal(1,1)	0.688	0.612	0.505	0.351	0.128
Beta(2,2)	0.981	0.967	0.938	0.868	0.622
Beta(2,3)	0.968	0.948	0.909	0.820	0.543

Table D.4 Power Study: Sample size 20, observations 16, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.096	0.048	0.010
Gamma(1.5)	0.939	0.905	0.846	0.723	0.409
Gamma(2.5)	0.982	0.970	0.944	0.879	0.649
Gamma(4.0)	0.992	0.986	0.971	0.929	0.762
Weibull(2.0)	0.993	0.988	0.976	0.943	0.779
Weibull(3.0)	0.998	0.997	0.992	0.979	0.892
Lognormal(0,2)	0.115	0.084	0.052	0.025	0.005
Lognormal(1,1)	0.820	0.761	0.667	0.515	0.235
Beta(2,2)	0.997	0.995	0.988	0.965	0.839
Beta(2,3)	0.994	0.990	0.977	0.942	0.777

Table D.5 Power Study: Sample size 25, observations 20, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.149	0.099	0.051	0.010
Gamma(1.5)	0.978	0.963	0.935	0.859	0.599
Gamma(2.5)	0.996	0.992	0.983	0.957	0.823
Gamma(4.0)	0.998	0.997	0.993	0.980	0.901
Weibull(2.0)	0.999	0.998	0.995	0.985	0.911
Weibull(3.0)	1.000	0.999	0.999	0.996	0.969
Lognormal(0,2)	0.114	0.083	0.053	0.026	0.005
Lognormal(1,1)	0.904	0.864	0.800	0.672	0.369
Beta(2,2)	1.000	0.999	0.998	0.992	0.946
Beta(2,3)	0.999	0.998	0.995	0.985	0.913

Table D.6 Power Study: Sample size 30, observations 24, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.151	0.101	0.050	0.010
Gamma(1.5)	0.993	0.986	0.973	0.930	0.747
Gamma(2.5)	0.999	0.998	0.995	0.984	0.919
Gamma(4.0)	1.000	0.999	0.999	0.995	0.963
Weibull(2.0)	1.000	0.999	0.999	0.996	0.969
Weibull(3.0)	1.000	1.000	1.000	0.999	0.992
Lognormal(0,2)	0.117	0.085	0.054	0.026	0.005
Lognormal(1,1)	0.951	0.926	0.884	0.785	0.521
Beta(2,2)	1.000	1.000	1.000	0.999	0.984
Beta(2,3)	1.000	1.000	0.999	0.996	0.969

Table D.7 Power Study: Sample size 35, observations 28, shape 0.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.151	0.101	0.052	0.011
Gamma(1.5)	0.997	0.995	0.989	0.970	0.847
Gamma(2.5)	1.000	1.000	0.999	0.996	0.966
Gamma(4.0)	1.000	1.000	1.000	0.999	0.987
Weibull(2.0)	1.000	1.000	1.000	0.999	0.989
Weibull(3.0)	1.000	1.000	1.000	1.000	0.998
Lognormal(0,2)	0.121	0.087	0.057	0.028	0.005
Lognormal(1,1)	0.976	0.961	0.934	0.867	0.637
Beta(2,2)	1.000	1.000	1.000	1.000	0.995
Beta(2,3)	1.000	1.000	1.000	0.999	0.989

Table D.8 Power Study: Sample size 5, observations 8, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.100	0.052	0.010
Gamma(1.5)	0.237	0.182	0.123	0.064	0.013
Gamma(2.5)	0.283	0.218	0.151	0.079	0.016
Gamma(4.0)	0.312	0.246	0.171	0.088	0.018
Weibull(2.0)	0.328	0.258	0.181	0.096	0.020
Weibull(3.0)	0.389	0.314	0.229	0.127	0.027
Lognormal(0,2)	0.095	0.069	0.045	0.022	0.004
Lognormal(1,1)	0.189	0.141	0.094	0.047	0.009
Beta(2,2)	0.376	0.300	0.216	0.118	0.026
Beta(2,3)	0.341	0.269	0.188	0.103	0.022

Table D.9 Power Study: Sample size 10, observations 8, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.050	0.010
Gamma(1.5)	0.307	0.240	0.167	0.090	0.019
Gamma(2.5)	0.426	0.347	0.259	0.151	0.038
Gamma(4.0)	0.511	0.432	0.338	0.211	0.060
Weibull(2.0)	0.551	0.469	0.369	0.235	0.071
Weibull(3.0)	0.683	0.610	0.513	0.364	0.138
Lognormal(0,2)	0.031	0.020	0.012	0.005	0.001
Lognormal(1,1)	0.191	0.145	0.098	0.050	0.009
Beta(2,2)	0.636	0.556	0.452	0.304	0.101
Beta(2,3)	0.561	0.479	0.376	0.240	0.073

Table D.10 Power Study: Sample size 15, observations 12, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.100	0.049	0.010
Gamma(1.5)	0.368	0.297	0.217	0.121	0.031
Gamma(2.5)	0.558	0.480	0.380	0.245	0.080
Gamma(4.0)	0.669	0.597	0.502	0.353	0.138
Weibull(2.0)	0.716	0.643	0.544	0.392	0.158
Weibull(3.0)	0.855	0.807	0.734	0.597	0.327
Lognormal(0,2)	0.012	0.008	0.004	0.002	0.000
Lognormal(1,1)	0.201	0.152	0.104	0.053	0.012
Beta(2,2)	0.810	0.749	0.661	0.508	0.236
Beta(2,3)	0.731	0.657	0.559	0.400	0.164

Table D.11 Power Study: Sample size 20, observations 16, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.148	0.099	0.051	0.010
Gamma(1.5)	0.419	0.345	0.257	0.156	0.042
Gamma(2.5)	0.660	0.584	0.484	0.345	0.132
Gamma(4.0)	0.787	0.727	0.641	0.504	0.241
Weibull(2.0)	0.826	0.769	0.687	0.546	0.268
Weibull(3.0)	0.934	0.905	0.858	0.769	0.522
Lognormal(0,2)	0.005	0.003	0.002	0.001	0.000
Lognormal(1,1)	0.205	0.155	0.105	0.056	0.012
Beta(2,2)	0.901	0.860	0.795	0.677	0.385
Beta(2,3)	0.835	0.781	0.694	0.554	0.271

Table D.12 Power Study: Sample size 25, observations 20, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.149	0.100	0.052	0.010
Gamma(1.5)	0.476	0.395	0.306	0.192	0.054
Gamma(2.5)	0.745	0.678	0.587	0.445	0.189
Gamma(4.0)	0.863	0.815	0.748	0.623	0.351
Weibull(2.0)	0.895	0.853	0.790	0.669	0.388
Weibull(3.0)	0.972	0.958	0.933	0.877	0.686
Lognormal(0,2)	0.002	0.002	0.001	0.000	0.000
Lognormal(1,1)	0.212	0.161	0.110	0.058	0.013
Beta(2,2)	0.951	0.927	0.886	0.799	0.536
Beta(2,3)	0.907	0.866	0.805	0.687	0.396

Table D.13 Power Study: Sample size 30, observations 24, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.100	0.050	0.010
Gamma(1.5)	0.522	0.440	0.344	0.224	0.070
Gamma(2.5)	0.812	0.754	0.672	0.532	0.266
Gamma(4.0)	0.914	0.879	0.826	0.724	0.466
Weibull(2.0)	0.938	0.910	0.864	0.770	0.511
Weibull(3.0)	0.989	0.982	0.969	0.937	0.808
Lognormal(0,2)	0.001	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.220	0.170	0.119	0.063	0.014
Beta(2,2)	0.978	0.964	0.938	0.878	0.673
Beta(2,3)	0.946	0.918	0.873	0.780	0.519

Table D.14 Power Study: Sample size 35, observations 28, shape 1.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.099	0.050	0.010
Gamma(1.5)	0.564	0.481	0.382	0.250	0.081
Gamma(2.5)	0.862	0.813	0.737	0.609	0.331
Gamma(4.0)	0.946	0.923	0.883	0.800	0.568
Weibull(2.0)	0.964	0.946	0.914	0.843	0.613
Weibull(3.0)	0.995	0.992	0.986	0.968	0.885
Lognormal(0,2)	0.001	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.232	0.178	0.124	0.069	0.015
Beta(2,2)	0.989	0.982	0.966	0.927	0.771
Beta(2,3)	0.969	0.951	0.918	0.849	0.617

Table D.15 Power Study: Sample size 5, observations 4, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.152	0.101	0.050	0.010
Gamma(1.5)	0.202	0.152	0.099	0.051	0.010
Gamma(2.5)	0.235	0.179	0.120	0.061	0.014
Gamma(4.0)	0.262	0.201	0.137	0.071	0.014
Weibull(2.0)	0.280	0.217	0.149	0.078	0.015
Weibull(3.0)	0.335	0.264	0.184	0.099	0.020
Lognormal(0,2)	0.079	0.057	0.038	0.019	0.004
Lognormal(1,1)	0.159	0.117	0.078	0.038	0.007
Beta(2,2)	0.322	0.253	0.177	0.095	0.021
Beta(2,3)	0.293	0.227	0.156	0.083	0.016

Table D.16 Power Study: Sample size 10, observations 8, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.145	0.096	0.048	0.009
Gamma(1.5)	0.200	0.149	0.099	0.049	0.009
Gamma(2.5)	0.292	0.229	0.159	0.085	0.019
Gamma(4.0)	0.373	0.301	0.220	0.127	0.030
Weibull(2.0)	0.408	0.331	0.245	0.142	0.036
Weibull(3.0)	0.553	0.474	0.377	0.246	0.076
Lognormal(0,2)	0.018	0.011	0.006	0.002	0.000
Lognormal(1,1)	0.115	0.082	0.053	0.024	0.004
Beta(2,2)	0.505	0.425	0.325	0.200	0.057
Beta(2,3)	0.423	0.345	0.257	0.152	0.038

Table D.17 Power Study: Sample size 15, observations 12, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.153	0.103	0.052	0.010
Gamma(1.5)	0.201	0.151	0.102	0.053	0.010
Gamma(2.5)	0.351	0.283	0.206	0.118	0.028
Gamma(4.0)	0.463	0.388	0.301	0.185	0.055
Weibull(2.0)	0.517	0.440	0.346	0.220	0.068
Weibull(3.0)	0.714	0.647	0.558	0.415	0.178
Lognormal(0,2)	0.004	0.002	0.001	0.000	0.000
Lognormal(1,1)	0.092	0.066	0.041	0.019	0.003
Beta(2,2)	0.642	0.567	0.465	0.321	0.113
Beta(2,3)	0.537	0.458	0.361	0.232	0.070

Table D.18 Power Study: Sample size 20, observations 16, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.152	0.101	0.050	0.010
Gamma(1.5)	0.205	0.154	0.103	0.051	0.011
Gamma(2.5)	0.400	0.327	0.242	0.142	0.037
Gamma(4.0)	0.550	0.475	0.379	0.250	0.087
Weibull(2.0)	0.605	0.527	0.424	0.286	0.101
Weibull(3.0)	0.820	0.767	0.685	0.549	0.287
Lognormal(0,2)	0.001	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.076	0.053	0.032	0.014	0.003
Beta(2,2)	0.751	0.684	0.585	0.431	0.183
Beta(2,3)	0.632	0.554	0.449	0.306	0.107

Table D.19 Power Study: Sample size 25, observations 20, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.153	0.102	0.050	0.010
Gamma(1.5)	0.199	0.151	0.101	0.049	0.010
Gamma(2.5)	0.439	0.365	0.278	0.168	0.049
Gamma(4.0)	0.621	0.549	0.452	0.312	0.116
Weibull(2.0)	0.679	0.607	0.510	0.363	0.144
Weibull(3.0)	0.889	0.851	0.789	0.671	0.408
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.064	0.045	0.027	0.012	0.002
Beta(2,2)	0.818	0.760	0.674	0.529	0.257
Beta(2,3)	0.699	0.626	0.527	0.374	0.147

Table D.20 Power Study: Sample size 30, observations 24, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.151	0.101	0.050	0.009
Gamma(1.5)	0.202	0.152	0.103	0.050	0.010
Gamma(2.5)	0.480	0.403	0.312	0.199	0.062
Gamma(4.0)	0.675	0.603	0.511	0.374	0.157
Weibull(2.0)	0.742	0.675	0.581	0.433	0.185
Weibull(3.0)	0.930	0.902	0.857	0.764	0.518
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.053	0.036	0.022	0.009	0.002
Beta(2,2)	0.876	0.829	0.759	0.627	0.338
Beta(2,3)	0.758	0.692	0.598	0.448	0.192

Table D.21 Power Study: Sample size 35, observations 28, shape 1.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.152	0.102	0.051	0.010
Gamma(1.5)	0.199	0.150	0.100	0.052	0.010
Gamma(2.5)	0.519	0.443	0.347	0.227	0.070
Gamma(4.0)	0.728	0.663	0.572	0.437	0.197
Weibull(2.0)	0.793	0.732	0.643	0.502	0.233
Weibull(3.0)	0.957	0.938	0.906	0.837	0.623
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.046	0.032	0.019	0.008	0.001
Beta(2,2)	0.911	0.875	0.813	0.702	0.414
Beta(2,3)	0.810	0.751	0.661	0.520	0.243

Table D.22 Power Study: Sample size 5, observations 4, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.148	0.099	0.050	0.009
Gamma(1.5)	0.181	0.134	0.088	0.044	0.009
Gamma(2.5)	0.212	0.159	0.107	0.054	0.011
Gamma(4.0)	0.239	0.180	0.122	0.062	0.012
Weibull(2.0)	0.252	0.192	0.131	0.068	0.013
Weibull(3.0)	0.306	0.239	0.168	0.090	0.018
Lognormal(0,2)	0.067	0.047	0.030	0.015	0.003
Lognormal(1,1)	0.138	0.100	0.065	0.032	0.006
Beta(2,2)	0.296	0.228	0.161	0.085	0.017
Beta(2,3)	0.263	0.201	0.140	0.071	0.014

Table D.23 Power Study: Sample size 10, observations 8, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.148	0.097	0.049	0.010
Gamma(1.5)	0.151	0.109	0.071	0.033	0.006
Gamma(2.5)	0.230	0.175	0.117	0.061	0.013
Gamma(4.0)	0.302	0.237	0.166	0.091	0.022
Weibull(2.0)	0.330	0.261	0.188	0.106	0.026
Weibull(3.0)	0.474	0.397	0.302	0.190	0.058
Lognormal(0,2)	0.013	0.008	0.004	0.002	0.000
Lognormal(1,1)	0.085	0.060	0.037	0.017	0.003
Beta(2,2)	0.428	0.349	0.259	0.154	0.042
Beta(2,3)	0.349	0.279	0.201	0.113	0.027

Table D.24 Power Study: Sample size 15, observations 12, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.049	0.010
Gamma(1.5)	0.133	0.096	0.062	0.029	0.005
Gamma(2.5)	0.250	0.192	0.134	0.070	0.016
Gamma(4.0)	0.352	0.284	0.209	0.118	0.030
Weibull(2.0)	0.401	0.326	0.243	0.140	0.036
Weibull(3.0)	0.610	0.535	0.441	0.306	0.113
Lognormal(0,2)	0.003	0.002	0.001	0.000	0.000
Lognormal(1,1)	0.058	0.040	0.023	0.010	0.001
Beta(2,2)	0.533	0.450	0.352	0.222	0.069
Beta(2,3)	0.425	0.348	0.261	0.151	0.038

Table D.25 Power Study: Sample size 20, observations 16, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.152	0.101	0.051	0.011
Gamma(1.5)	0.124	0.090	0.056	0.025	0.004
Gamma(2.5)	0.272	0.213	0.149	0.080	0.018
Gamma(4.0)	0.404	0.333	0.250	0.151	0.041
Weibull(2.0)	0.463	0.387	0.291	0.180	0.050
Weibull(3.0)	0.717	0.650	0.556	0.415	0.180
Lognormal(0,2)	0.001	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.042	0.028	0.016	0.007	0.001
Beta(2,2)	0.626	0.548	0.445	0.301	0.103
Beta(2,3)	0.491	0.412	0.314	0.195	0.057

Table D.26 Power Study: Sample size 25, observations 20, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.152	0.101	0.053	0.010
Gamma(1.5)	0.109	0.078	0.047	0.021	0.003
Gamma(2.5)	0.283	0.222	0.155	0.084	0.019
Gamma(4.0)	0.448	0.374	0.284	0.177	0.051
Weibull(2.0)	0.512	0.433	0.339	0.219	0.067
Weibull(3.0)	0.791	0.734	0.648	0.512	0.255
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.030	0.020	0.011	0.005	0.000
Beta(2,2)	0.687	0.614	0.514	0.365	0.142
Beta(2,3)	0.540	0.460	0.359	0.231	0.070

Table D.27 Power Study: Sample size 30, observations 24, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.204	0.152	0.102	0.051	0.009
Gamma(1.5)	0.101	0.070	0.044	0.020	0.003
Gamma(2.5)	0.296	0.232	0.163	0.092	0.021
Gamma(4.0)	0.488	0.409	0.318	0.204	0.061
Weibull(2.0)	0.563	0.481	0.382	0.252	0.079
Weibull(3.0)	0.846	0.795	0.724	0.596	0.327
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.022	0.014	0.008	0.003	0.000
Beta(2,2)	0.746	0.674	0.580	0.430	0.176
Beta(2,3)	0.577	0.494	0.396	0.262	0.084

Table D.28 Power Study: Sample size 35, observations 28, shape 2.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.152	0.101	0.051	0.009
Gamma(1.5)	0.092	0.064	0.039	0.017	0.003
Gamma(2.5)	0.312	0.245	0.175	0.097	0.023
Gamma(4.0)	0.526	0.450	0.357	0.237	0.079
Weibull(2.0)	0.605	0.525	0.425	0.291	0.104
Weibull(3.0)	0.889	0.851	0.791	0.679	0.413
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.016	0.010	0.006	0.002	0.000
Beta(2,2)	0.796	0.733	0.643	0.497	0.229
Beta(2,3)	0.633	0.554	0.454	0.310	0.111

Table D.29 Power Study: Sample size 5, observations 4, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.151	0.101	0.049	0.010
Gamma(1.5)	0.165	0.121	0.080	0.039	0.008
Gamma(2.5)	0.198	0.148	0.099	0.049	0.010
Gamma(4.0)	0.218	0.165	0.110	0.054	0.011
Weibull(2.0)	0.236	0.182	0.123	0.061	0.013
Weibull(3.0)	0.287	0.222	0.154	0.079	0.017
Lognormal(0,2)	0.065	0.046	0.031	0.014	0.003
Lognormal(1,1)	0.131	0.096	0.062	0.029	0.006
Beta(2,2)	0.278	0.215	0.149	0.078	0.018
Beta(2,3)	0.245	0.185	0.128	0.064	0.014

Table D.30 Power Study: Sample size 10, observations 8, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.151	0.101	0.051	0.009
Gamma(1.5)	0.127	0.090	0.058	0.027	0.004
Gamma(2.5)	0.197	0.147	0.098	0.050	0.009
Gamma(4.0)	0.261	0.203	0.140	0.076	0.016
Weibull(2.0)	0.289	0.225	0.160	0.088	0.019
Weibull(3.0)	0.425	0.350	0.265	0.161	0.043
Lognormal(0,2)	0.010	0.007	0.003	0.001	0.000
Lognormal(1,1)	0.070	0.050	0.030	0.013	0.002
Beta(2,2)	0.377	0.303	0.221	0.128	0.031
Beta(2,3)	0.306	0.240	0.170	0.093	0.020

Table D.31 Power Study: Sample size 15, observations 12, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.195	0.148	0.096	0.048	0.010
Gamma(1.5)	0.102	0.073	0.046	0.021	0.003
Gamma(2.5)	0.200	0.149	0.099	0.048	0.010
Gamma(4.0)	0.288	0.228	0.159	0.086	0.020
Weibull(2.0)	0.332	0.264	0.187	0.102	0.025
Weibull(3.0)	0.543	0.464	0.368	0.241	0.081
Lognormal(0,2)	0.002	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.042	0.026	0.015	0.006	0.001
Beta(2,2)	0.463	0.384	0.287	0.170	0.046
Beta(2,3)	0.353	0.282	0.202	0.114	0.026

Table D.32 Power Study: Sample size 20, observations 16, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.151	0.100	0.049	0.009
Gamma(1.5)	0.085	0.059	0.035	0.013	0.002
Gamma(2.5)	0.203	0.155	0.103	0.051	0.009
Gamma(4.0)	0.318	0.255	0.183	0.100	0.024
Weibull(2.0)	0.368	0.296	0.217	0.122	0.030
Weibull(3.0)	0.633	0.559	0.463	0.321	0.121
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.026	0.017	0.010	0.004	0.000
Beta(2,2)	0.529	0.448	0.349	0.219	0.066
Beta(2,3)	0.399	0.323	0.238	0.136	0.034

Table D.33 Power Study: Sample size 25, observations 20, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.153	0.102	0.051	0.010
Gamma(1.5)	0.071	0.047	0.028	0.011	0.002
Gamma(2.5)	0.202	0.152	0.101	0.052	0.009
Gamma(4.0)	0.344	0.273	0.199	0.115	0.028
Weibull(2.0)	0.408	0.333	0.245	0.146	0.039
Weibull(3.0)	0.711	0.642	0.547	0.404	0.173
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.018	0.011	0.006	0.003	0.000
Beta(2,2)	0.590	0.509	0.406	0.272	0.091
Beta(2,3)	0.436	0.355	0.266	0.157	0.042

Table D.34 Power Study: Sample size 30, observations 24, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.204	0.151	0.101	0.050	0.010
Gamma(1.5)	0.061	0.041	0.024	0.009	0.001
Gamma(2.5)	0.200	0.148	0.099	0.049	0.009
Gamma(4.0)	0.368	0.297	0.216	0.125	0.034
Weibull(2.0)	0.436	0.357	0.268	0.161	0.043
Weibull(3.0)	0.762	0.698	0.609	0.467	0.223
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.011	0.007	0.003	0.001	0.000
Beta(2,2)	0.646	0.567	0.462	0.314	0.110
Beta(2,3)	0.460	0.380	0.285	0.169	0.046

Table D.35 Power Study: Sample size 35, observations 28, shape 2.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.150	0.102	0.051	0.010
Gamma(1.5)	0.053	0.035	0.020	0.008	0.001
Gamma(2.5)	0.200	0.149	0.100	0.050	0.009
Gamma(4.0)	0.390	0.317	0.236	0.141	0.038
Weibull(2.0)	0.464	0.385	0.293	0.178	0.052
Weibull(3.0)	0.811	0.755	0.677	0.541	0.274
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.008	0.005	0.003	0.001	0.000
Beta(2,2)	0.686	0.608	0.507	0.357	0.132
Beta(2,3)	0.496	0.414	0.319	0.196	0.056

Table D.36 Power Study: Sample size 5, observations 4, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.148	0.098	0.048	0.010
Gamma(1.5)	0.159	0.116	0.076	0.038	0.007
Gamma(2.5)	0.187	0.141	0.092	0.046	0.010
Gamma(4.0)	0.206	0.156	0.105	0.052	0.011
Weibull(2.0)	0.225	0.172	0.115	0.057	0.011
Weibull(3.0)	0.275	0.211	0.144	0.074	0.016
Lognormal(0,2)	0.058	0.041	0.026	0.012	0.002
Lognormal(1,1)	0.121	0.087	0.056	0.027	0.005
Beta(2,2)	0.263	0.203	0.140	0.072	0.015
Beta(2,3)	0.238	0.180	0.122	0.061	0.012

Table D.37 Power Study: Sample size 10, observations 8, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.152	0.100	0.050	0.010
Gamma(1.5)	0.110	0.081	0.050	0.022	0.004
Gamma(2.5)	0.176	0.131	0.085	0.040	0.008
Gamma(4.0)	0.234	0.179	0.121	0.061	0.013
Weibull(2.0)	0.263	0.204	0.139	0.074	0.017
Weibull(3.0)	0.393	0.323	0.237	0.139	0.037
Lognormal(0,2)	0.010	0.006	0.003	0.001	0.000
Lognormal(1,1)	0.061	0.042	0.024	0.010	0.001
Beta(2,2)	0.346	0.276	0.195	0.110	0.026
Beta(2,3)	0.278	0.216	0.147	0.078	0.016

Table D.38 Power Study: Sample size 15, observations 12, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.196	0.148	0.101	0.051	0.011
Gamma(1.5)	0.083	0.057	0.035	0.015	0.002
Gamma(2.5)	0.168	0.124	0.082	0.040	0.007
Gamma(4.0)	0.246	0.188	0.131	0.069	0.016
Weibull(2.0)	0.289	0.225	0.158	0.084	0.020
Weibull(3.0)	0.493	0.416	0.326	0.208	0.065
Lognormal(0,2)	0.002	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.033	0.022	0.013	0.005	0.001
Beta(2,2)	0.411	0.332	0.248	0.145	0.040
Beta(2,3)	0.310	0.241	0.171	0.091	0.021

Table D.39 Power Study: Sample size 20, observations 16, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.152	0.101	0.051	0.011
Gamma(1.5)	0.066	0.045	0.026	0.011	0.002
Gamma(2.5)	0.162	0.119	0.076	0.035	0.007
Gamma(4.0)	0.266	0.208	0.145	0.076	0.017
Weibull(2.0)	0.308	0.242	0.170	0.093	0.023
Weibull(3.0)	0.568	0.492	0.393	0.262	0.095
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.020	0.013	0.006	0.002	0.000
Beta(2,2)	0.467	0.386	0.292	0.173	0.051
Beta(2,3)	0.339	0.270	0.192	0.104	0.025

Table D.40 Power Study: Sample size 25, observations 20, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.099	0.050	0.011
Gamma(1.5)	0.052	0.034	0.020	0.008	0.001
Gamma(2.5)	0.152	0.110	0.070	0.034	0.006
Gamma(4.0)	0.273	0.214	0.149	0.080	0.019
Weibull(2.0)	0.332	0.262	0.187	0.105	0.026
Weibull(3.0)	0.640	0.564	0.466	0.328	0.126
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.012	0.007	0.004	0.002	0.000
Beta(2,2)	0.515	0.433	0.333	0.209	0.062
Beta(2,3)	0.361	0.288	0.205	0.113	0.027

Table D.41 Power Study: Sample size 30, observations 24, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.151	0.101	0.051	0.010
Gamma(1.5)	0.041	0.028	0.016	0.006	0.001
Gamma(2.5)	0.147	0.109	0.070	0.033	0.006
Gamma(4.0)	0.289	0.228	0.162	0.090	0.022
Weibull(2.0)	0.349	0.282	0.205	0.113	0.028
Weibull(3.0)	0.688	0.622	0.528	0.388	0.165
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.007	0.004	0.002	0.001	0.000
Beta(2,2)	0.559	0.478	0.378	0.242	0.078
Beta(2,3)	0.378	0.305	0.223	0.123	0.030

Table D.42 Power Study: Sample size 35, observations 28, shape 3.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.202	0.152	0.102	0.051	0.010
Gamma(1.5)	0.034	0.022	0.011	0.004	0.000
Gamma(2.5)	0.142	0.103	0.065	0.031	0.005
Gamma(4.0)	0.301	0.238	0.170	0.094	0.022
Weibull(2.0)	0.369	0.299	0.220	0.126	0.031
Weibull(3.0)	0.742	0.678	0.589	0.446	0.203
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.005	0.003	0.002	0.001	0.000
Beta(2,2)	0.596	0.513	0.413	0.273	0.089
Beta(2,3)	0.402	0.326	0.241	0.142	0.035

Table D.43 Power Study: Sample size 5, observations 4, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.049	0.010
Gamma(1.5)	0.153	0.112	0.074	0.037	0.007
Gamma(2.5)	0.180	0.135	0.091	0.045	0.009
Gamma(4.0)	0.200	0.149	0.099	0.050	0.009
Weibull(2.0)	0.216	0.163	0.110	0.055	0.011
Weibull(3.0)	0.266	0.205	0.144	0.075	0.014
Lognormal(0,2)	0.056	0.039	0.025	0.012	0.002
Lognormal(1,1)	0.117	0.086	0.055	0.026	0.005
Beta(2,2)	0.255	0.196	0.136	0.070	0.015
Beta(2,3)	0.226	0.171	0.116	0.058	0.011

Table D.44 Power Study: Sample size 10, observations 8, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.154	0.102	0.049	0.010
Gamma(1.5)	0.101	0.071	0.043	0.019	0.003
Gamma(2.5)	0.160	0.117	0.077	0.035	0.006
Gamma(4.0)	0.215	0.164	0.111	0.056	0.011
Weibull(2.0)	0.242	0.187	0.127	0.066	0.014
Weibull(3.0)	0.367	0.298	0.217	0.123	0.032
Lognormal(0,2)	0.008	0.005	0.003	0.001	0.000
Lognormal(1,1)	0.055	0.037	0.022	0.009	0.001
Beta(2,2)	0.328	0.259	0.182	0.098	0.024
Beta(2,3)	0.256	0.196	0.134	0.070	0.014

Table D.45 Power Study: Sample size 15, observations 12, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.149	0.099	0.050	0.009
Gamma(1.5)	0.072	0.049	0.029	0.012	0.001
Gamma(2.5)	0.146	0.106	0.068	0.031	0.005
Gamma(4.0)	0.220	0.165	0.112	0.056	0.011
Weibull(2.0)	0.257	0.198	0.136	0.070	0.014
Weibull(3.0)	0.456	0.380	0.290	0.179	0.053
Lognormal(0,2)	0.001	0.001	0.001	0.000	0.000
Lognormal(1,1)	0.028	0.019	0.010	0.004	0.000
Beta(2,2)	0.373	0.299	0.216	0.122	0.031
Beta(2,3)	0.278	0.214	0.146	0.075	0.016

Table D.46 Power Study: Sample size 20, observations 16, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.197	0.147	0.098	0.048	0.009
Gamma(1.5)	0.053	0.035	0.020	0.008	0.001
Gamma(2.5)	0.136	0.097	0.062	0.028	0.005
Gamma(4.0)	0.224	0.169	0.117	0.060	0.012
Weibull(2.0)	0.266	0.206	0.142	0.075	0.016
Weibull(3.0)	0.521	0.444	0.349	0.223	0.069
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.017	0.010	0.006	0.002	0.000
Beta(2,2)	0.417	0.338	0.250	0.145	0.036
Beta(2,3)	0.297	0.229	0.159	0.084	0.018

Table D.47 Power Study: Sample size 25, observations 20, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.152	0.100	0.051	0.009
Gamma(1.5)	0.040	0.026	0.014	0.005	0.001
Gamma(2.5)	0.123	0.089	0.055	0.025	0.004
Gamma(4.0)	0.231	0.176	0.119	0.061	0.012
Weibull(2.0)	0.283	0.219	0.151	0.081	0.017
Weibull(3.0)	0.585	0.509	0.412	0.277	0.095
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.009	0.005	0.003	0.001	0.000
Beta(2,2)	0.459	0.380	0.287	0.171	0.046
Beta(2,3)	0.310	0.240	0.167	0.089	0.019

Table D.48 Power Study: Sample size 30, observations 24, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.153	0.103	0.052	0.011
Gamma(1.5)	0.031	0.020	0.011	0.004	0.001
Gamma(2.5)	0.116	0.081	0.052	0.023	0.003
Gamma(4.0)	0.235	0.179	0.124	0.066	0.013
Weibull(2.0)	0.295	0.228	0.161	0.087	0.019
Weibull(3.0)	0.638	0.562	0.466	0.329	0.124
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.005	0.003	0.002	0.001	0.000
Beta(2,2)	0.501	0.419	0.323	0.198	0.056
Beta(2,3)	0.321	0.250	0.177	0.097	0.021

Table D.49 Power Study: Sample size 35, observations 28, shape 3.5

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.203	0.153	0.102	0.051	0.010
Gamma(1.5)	0.025	0.015	0.008	0.003	0.000
Gamma(2.5)	0.110	0.077	0.047	0.021	0.004
Gamma(4.0)	0.241	0.185	0.127	0.066	0.015
Weibull(2.0)	0.309	0.241	0.169	0.092	0.022
Weibull(3.0)	0.683	0.612	0.514	0.369	0.154
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.004	0.002	0.001	0.000	0.000
Beta(2,2)	0.530	0.446	0.346	0.216	0.065
Beta(2,3)	0.337	0.266	0.189	0.102	0.024

Table D.50 Power Study: Sample size 5, observations 4, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.152	0.102	0.049	0.010
Gamma(1.5)	0.151	0.112	0.072	0.034	0.007
Gamma(2.5)	0.176	0.132	0.088	0.042	0.008
Gamma(4.0)	0.195	0.147	0.097	0.046	0.009
Weibull(2.0)	0.212	0.160	0.106	0.052	0.010
Weibull(3.0)	0.260	0.203	0.139	0.071	0.014
Lognormal(0,2)	0.054	0.038	0.024	0.011	0.002
Lognormal(1,1)	0.114	0.084	0.053	0.025	0.005
Beta(2,2)	0.249	0.192	0.131	0.067	0.013
Beta(2,3)	0.218	0.166	0.111	0.055	0.011

Table D.51 Power Study: Sample size 10, observations 8, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.200	0.148	0.099	0.049	0.009
Gamma(1.5)	0.094	0.065	0.039	0.017	0.002
Gamma(2.5)	0.149	0.106	0.069	0.032	0.005
Gamma(4.0)	0.200	0.150	0.100	0.049	0.009
Weibull(2.0)	0.226	0.171	0.114	0.057	0.012
Weibull(3.0)	0.352	0.279	0.200	0.112	0.026
Lognormal(0,2)	0.007	0.005	0.002	0.001	0.000
Lognormal(1,1)	0.052	0.034	0.020	0.008	0.001
Beta(2,2)	0.308	0.240	0.166	0.089	0.019
Beta(2,3)	0.241	0.181	0.123	0.062	0.011

Table D.52 Power Study: Sample size 15, observations 12, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.198	0.149	0.099	0.049	0.009
Gamma(1.5)	0.063	0.043	0.025	0.010	0.002
Gamma(2.5)	0.130	0.094	0.059	0.027	0.004
Gamma(4.0)	0.200	0.151	0.100	0.049	0.009
Weibull(2.0)	0.234	0.178	0.120	0.060	0.011
Weibull(3.0)	0.425	0.351	0.264	0.159	0.043
Lognormal(0,2)	0.001	0.001	0.000	0.000	0.000
Lognormal(1,1)	0.025	0.015	0.009	0.003	0.000
Beta(2,2)	0.349	0.279	0.198	0.107	0.023
Beta(2,3)	0.253	0.194	0.131	0.066	0.011

Table D.53 Power Study: Sample size 20, observations 16, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.196	0.147	0.098	0.048	0.009
Gamma(1.5)	0.044	0.028	0.015	0.006	0.001
Gamma(2.5)	0.116	0.082	0.052	0.022	0.004
Gamma(4.0)	0.201	0.150	0.100	0.050	0.009
Weibull(2.0)	0.241	0.182	0.124	0.062	0.013
Weibull(3.0)	0.481	0.403	0.313	0.195	0.059
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.013	0.008	0.004	0.002	0.000
Beta(2,2)	0.383	0.307	0.223	0.125	0.031
Beta(2,3)	0.267	0.203	0.140	0.070	0.014

Table D.54 Power Study: Sample size 25, observations 20, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.150	0.100	0.050	0.010
Gamma(1.5)	0.034	0.022	0.012	0.004	0.001
Gamma(2.5)	0.104	0.072	0.045	0.020	0.003
Gamma(4.0)	0.198	0.148	0.099	0.050	0.009
Weibull(2.0)	0.248	0.189	0.130	0.067	0.013
Weibull(3.0)	0.539	0.461	0.367	0.238	0.079
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.007	0.004	0.002	0.001	0.000
Beta(2,2)	0.417	0.340	0.252	0.145	0.039
Beta(2,3)	0.273	0.209	0.142	0.073	0.015

Table D.55 Power Study: Sample size 30, observations 24, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.199	0.148	0.098	0.050	0.011
Gamma(1.5)	0.024	0.015	0.008	0.003	0.000
Gamma(2.5)	0.094	0.066	0.040	0.017	0.003
Gamma(4.0)	0.203	0.152	0.101	0.051	0.010
Weibull(2.0)	0.256	0.194	0.134	0.068	0.014
Weibull(3.0)	0.588	0.511	0.415	0.281	0.099
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.004	0.002	0.001	0.000	0.000
Beta(2,2)	0.443	0.366	0.276	0.162	0.044
Beta(2,3)	0.278	0.212	0.146	0.075	0.016

Table D.56 Power Study: Sample size 35, observations 28, shape 4.0

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Original	0.201	0.151	0.100	0.050	0.010
Gamma(1.5)	0.018	0.011	0.006	0.002	0.000
Gamma(2.5)	0.086	0.059	0.036	0.015	0.002
Gamma(4.0)	0.202	0.153	0.102	0.050	0.011
Weibull(2.0)	0.260	0.201	0.137	0.070	0.014
Weibull(3.0)	0.629	0.554	0.455	0.316	0.119
Lognormal(0,2)	0.000	0.000	0.000	0.000	0.000
Lognormal(1,1)	0.003	0.001	0.001	0.000	0.000
Beta(2,2)	0.472	0.394	0.298	0.179	0.051
Beta(2,3)	0.289	0.224	0.155	0.081	0.017

Appendix E. Fortran Program for the Calculation of Critical Values

```
      PROGRAM ZCRITIC
*     THIS PROGRAMS GENERATES THE CRITICAL VALUES
*     FOR THE GAMMA DISTRIBUTION WITH SHAPE PARAMETER=0.5
*     FOR OTHER SHAPES CHANGE K=0.5, AND INCLUDE OPEN05.F
*     INCLUDED FILE OPEN05.F CONTAINS OPEN STATEMENTS
*     TO OPEN TEX OUTPUT FILES, AND MUDIF INPUT FILES

      INTEGER S,I,J,II,JJ,SS,N,SEED,PASS,IERROR,LK,KL,LM
      REAL CRIT80,CRIT85,CRIT90,CRIT95,CRIT99,K
      REAL CRT80,CRT85,CRT90,CRT95,CRT99
      REAL Y(40),X(40),GAP(40),MUDIF(39)
      REAL XX(40),ZSTAR(0:10003),G(40)
      REAL NUMSUM,NUM,DENSUM,DENOM

      EXTERNAL RNSET,SVRGN,RNGAM
      INCLUDE 'zcr/include/open05.f'

      S=10000
      K=0.5
      KL=10*K+1
      LK=KL+1
      LM=LK+1
      WRITE(LK,970)
 970  FORMAT('\\begin{table}[h]'/'\\caption{Critical values for '
     +'$Z^*$ test statistic: Sample size N, shape parameter 0.5}')
      WRITE(LK,982)
 982  FORMAT('\\begin{center}')
      WRITE(LK,971)
 971   FORMAT('\\begin{tabular}{||c|c|c|c|c|c|c||} \\hline')
      WRITE(LK,983)
 983  FORMAT('Sample size&shape&$\\alpha=0.20$&$\\alpha=0.15$&$\\alpha'
     +'=0.10$&$\\alpha=0.05$&$\\alpha=0.01$\\zline',/,'\\hline')

       DO 2000 N=5,40,5
          CRT80=0.0
          CRT85=0.0
          CRT90=0.0
          CRT95=0.0
          CRT99=0.0
```

```
      DO 1999 PASS=1,10
       GO TO (15,16,17,18,19,20,21,22,23,24),PASS
15          SEED=23741324
                 GO TO 25
16          SEED=27465531
                 GO TO 25
17          SEED=59351236
                 GO TO 25
18          SEED=74639465
                 GO TO 25
19          SEED=56739493
                 GO TO 25
20          SEED=98423732
                 GO TO 25
21          SEED=56384628
                 GO TO 25
22          SEED=48897742
                 GO TO 25
23          SEED=96745663
                 GO TO 25
24          SEED=87346239
25          CALL RNSET(SEED)
                   I=0
                   J=0
                  II=0
                  JJ=0
                  SS=0
            DO 5 I=1,N
              X(I)=0.0
              Y(I)=0.0
            GAP(I)=0.0
              G(I)=0.0
5           CONTINUE
            DO 6 I=0,S+1
            ZSTAR(I)=0.0
6           CONTINUE
                     CRIT80=0.0
                     CRIT85=0.0
                     CRIT90=0.0
                     CRIT95=0.0
                     CRIT99=0.0
      REWIND (N)
                         READ(N,*)(MUDIF(I),I=1,N-1)
```

```
                        WRITE(LM,903)(MUDIF(I),I=1,N-1)
903                FORMAT(1(5F10.6))
                        DO 110 J=1,S
                CALL RNGAM(N,K,Y)
                        DO 7 I=1,N
                         X(I)=Y(I)+10.0
  7                     CONTINUE
                        CALL SVRGN(N,X,XX)
                        DO 8 I=1,N-1
                         GAP(I)=XX(I+1)-XX(I)
  8                     CONTINUE
                        DO 9 I=1,N-1
                         G(I)=GAP(I)/MUDIF(I)
  9                     CONTINUE
                NUMSUM=0.0
                        DO 10 I=1,N-2
                         NUMSUM=NUMSUM+(N-1-I)*G(I)
 10                     CONTINUE
                        NUM=2.0*NUMSUM
                        DENSUM=0.0
                        DO 11 I=1,N-1
                         DENSUM=DENSUM+G(I)
 11                     CONTINUE
                         DENOM=(N-2)*DENSUM
                         ZSTAR(J)=NUM/DENOM
 110                   CONTINUE
               SS=S+1
               CALL SVRGN(SS,ZSTAR,ZSTAR)
               CALL EXTRA(S,ZSTAR)
               CALL VALUES(ZSTAR,CRIT80,CRIT85,CRIT90,
     +                                CRIT95,CRIT99,SS)
              CRT80=CRT80+CRIT80
              CRT85=CRT85+CRIT85
              CRT90=CRT90+CRIT90
              CRT95=CRT95+CRIT95
              CRT99=CRT99+CRIT99
1999    CONTINUE
C             AVERAGING THE FIVE INDEPENDENT VALUES
              CRT80=CRT80/10
              CRT85=CRT85/10
              CRT90=CRT90/10
              CRT95=CRT95/10
              CRT99=CRT99/10
```

```
              WRITE(KL,902)CRT80,CRT85,CRT90,
     +                               CRT95,CRT99
902             FORMAT(1X,5F7.4)
                WRITE(LK,972)N,K,CRT80,CRT85,CRT90,
     +          CRT95,CRT99
972             FORMAT(I2,' & ',F3.1,5(' & ',F6.4),'\\','\\\\hline')
       CLOSE (N)
       S=10000
2000   CONTINUE
3000   CONTINUE
        WRITE(LK,973)
 973    FORMAT('\\end{tabular}'/'\\end{center}'/'\\end{table}')
        CLOSE(LK)
        CLOSE(LM)
STOP
1000   WRITE(*,1001) IERROR
1001   FORMAT(1X,'IOSTAT = ', IERROR/)
       END
***************************************************************
*               SUBROUTINE  CRIT                              *
*    USING THE TECHNIQUE EXPLAINED IN CHAPTER 3, FIND THE     *
*    CRITICAL VALUES                                          *
*                                                             *
***************************************************************
      SUBROUTINE CRIT(Y1,Y2,D1,D2,Y,RES)
      REAL M,B,Y1,Y2,D1,D2,Y,RES
      IF((D2-D1).EQ.0.0)D2 = D2 * 1.00001
      M = (Y2-Y1)/(D2-D1)
      B = Y1 - M*D1
      RES = (Y-B)/M
      RETURN
      END
*******************************************************************
*       SUBROUTINE  EXTRA                       *
*    THIS SUBROUTINE EXTRAPOLATES THE ZCRITICAL(I) DATA           *
*    TO GENERATE ZCRITICAL(0) AND ZCRITICAL(S+1) FOR COMPUTATION *
*    OF THE FIVE CRITICAL VALUES.                                 *
*                                                                 *
*******************************************************************
      SUBROUTINE EXTRA(N,D)
      INTEGER N,NO,N1
      REAL Y1,Y2,D(0:10003),D1,D2,ZZ
      Y1 = 0.5/N
```

```
      Y2 = 1.5/N
      D1 = D(1)
      D2 = D(2)
      CALL CRIT(Y1,Y2,D1,D2,0.0,ZZ)
      IF(ZZ.GE.0.0) THEN
        D(0) = ZZ
      ELSE
        D(0) = 0.0
      ENDIF
      Y1 = (REAL(N) - 1.5)/N
      Y2 = (REAL(N) - 0.5)/N
      N0 = N-1
      D1 = D(N0)
      D2 = D(N)
      CALL CRIT(Y1,Y2,D1,D2,1.0,ZZ)
      N1 = N + 1
      D(N1) = ZZ
      RETURN
      END
***************************************************************
*       *
*     THE FOLLOWING SUB DETERMINES THE %TILES AND FINDS        *
*     THE CRITICAL VALUES BY EVOKING THE SUBROUTINE CRIT       *
*       *
***************************************************************
      SUBROUTINE VALUES(D,CRIT80,CRIT85,CRIT90,CRIT95,CRIT99,N)
      INTEGER I,N,NN
      REAL D(0:10003),Y(0:10003),C80,C90,C95,C99,C85,
     + Y79,D79,Y81,D81,DIF90,Y89,Y91,D89,D91,DIF95,DIF80,
     + Y94,Y96,D94,D96,DIF99,Y98,Y100,D98,D100,DIF85,
     + Y84,D84,Y86,D86,CRIT85,CRIT80,CRIT90,CRIT95,CRIT99
      DO 100 I = 1,N
        Y(I) = (REAL(I) - 0.5)/REAL(N)
 100    CONTINUE
      Y(0) = 0.0
         NN = N + 1
         Y(NN) = 1.0
      C80 = 1000.0
      C85 = 1000.0
      C90 = 1000.0
      C95 = 1000.0
      C99 = 1000.0
      DO 200 I = NN,0,-1
```

```
      IF (Y(I).LE.0.75) GO TO 300
      IF (Y(I).GT.0.75.AND.Y(I).LE.0.80) THEN

C       GET THE DESIRED %TILE AT 80%
        DIF80 = .80 - Y(I)
        IF (DIF80.LE.C80) THEN
          C80 = DIF80
          Y79 = Y(I)
          D79 = D(I)
          Y81 = Y(I+1)
          D81 = D(I+1)
        ENDIF
      ELSEIF (Y(I).GT.0.80.AND.Y(I).LE.0.85) THEN
C       GET THE DESIRED %TILE AT 85%
        DIF85 = .85 - Y(I)
        IF (DIF85.LE.C85) THEN
          C85 = DIF85
          Y84 = Y(I)
          D84 = D(I)
          Y86 = Y(I+1)
          D86 = D(I+1)
        ENDIF
      ELSEIF (Y(I).GT.0.85.AND.Y(I).LE.0.90) THEN
C       GET THE DESIRED %TILE AT 90%
        DIF90 = .90 - Y(I)
        IF (DIF90.LE.C90) THEN
          C90 = DIF90
          Y89 = Y(I)
          D89 = D(I)
          Y91 = Y(I+1)
          D91 = D(I+1)
        ENDIF
      ELSEIF (Y(I).GT.0.90.AND.Y(I).LE.0.95) THEN
C       GET THE DESIRED %TILE AT 95%
        DIF95 = .95 - Y(I)
        IF (DIF95.LE.C95) THEN
          C95 = DIF95
          Y94 = Y(I)
          D94 = D(I)
          Y96 = Y(I+1)
          D96 = D(I+1)
        ENDIF
      ELSEIF (Y(I).GT.0.95.AND.Y(I).LE.0.99) THEN
```

```
C          GET THE DESIRED %TILE AT 99%
           DIF99 = .99 - Y(I)
           IF (DIF99.LE.C99) THEN
             C99 = DIF99
             Y98 = Y(I)
             D98 = D(I)
             Y100 = Y(I+1)
             D100 = D(I+1)
           ENDIF
         ENDIF
200    CONTINUE
300    IF (DIF80.EQ.0.0) THEN
         CRIT80 = D79
       ELSE
C        COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .20
         CALL CRIT(Y79,Y81,D79,D81,.80,CRIT80)
       ENDIF
       IF (DIF85.EQ.0.0) THEN
         CRIT85 = D84
       ELSE
C        COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .15
         CALL CRIT(Y84,Y86,D84,D86,.85,CRIT85)
       ENDIF
       IF (DIF90.EQ.0.0) THEN
         CRIT90 = D89
       ELSE
C        COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .10
         CALL CRIT(Y89,Y91,D89,D91,.90,CRIT90)
       ENDIF
       IF (DIF95.EQ.0.0) THEN
         CRIT95 = D94
       ELSE
C        COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .05
         CALL CRIT(Y94,Y96,D94,D96,.95,CRIT95)
       ENDIF
       IF (DIF99.EQ.0.0) THEN
         CRIT99 = D98
       ELSE
C        COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .01
         CALL CRIT(Y98,Y100,D98,D100,.99,CRIT99)
       ENDIF
       RETURN
       END
```

Appendix F. Fortran Program for the Power Study

```
      PROGRAM POWER
*     THIS PROGRAM GENERATES POWER TABLES IN LATEX FORMAT
*     IT RUNS FOR SHAPE PARAMETER=0.5. FOR OTHER SHAPES
*     CHANGE K=0.5, AND NECESSARY INCLUDE STATEMENTS THAT
*     OPEN MUDIF DATA FILES AND CRITICAL VALUE *.OUT FILES

      INTEGER I,J,IREPS,N,SEED,IERROR,NN
      INTEGER REJ(1:10,0:5),DIST,NUMDIST,II,JJ
      CHARACTER*14 D(10)
      REAL Y(35),X(35),GAP(35),MUDIF(34)
      REAL XX(35),ZSTAR(1:50005),G(20)
      REAL NUMSUM,NUM,DENSUM,DENOM,K
      REAL CRIT(35)
      REAL PWR(10,5)
      EXTERNAL RNSET,SVRGN,RNGAM,RNLNL
      EXTERNAL RNUN,RNBET,RNNOR,RNWIB
      INCLUDE 'zcr/include/open05.f'
      INCLUDE 'power/include/pow05.inc'
      IREPS=50000
      NUMDIST=10
      CENSOR=1.0
      K=0.5
      KL=10*K+1
      READ(KL,*)(CRIT(I),I=1,35)
      SEED=816283532
      CALL RNSET(SEED)

*     IF THE FOLLOWING SET OF ALTERNATIVE DISTRIBUTIONS
*     ARE CHANGED THEN EXTERNAL STATEMENT AT THE BEGINNING
*     SHOULD ALSO INCLUDE ADDED DISTRIBUTIONS
      D(1)="Original"
      D(2)="Gamma(1.5)"
      D(3)="Gamma(2.5)"
      D(4)="Gamma(4.0)"
      D(5)="Weibull(2.0)"
      D(6)="Weibull(3.0)"
      D(7)="Lognormal(0,2)"
      D(8)="Lognormal(1,1)"
      D(9)="Beta(2,2)"
      D(10)="Beta(2,3)"
      DO 2000  N=5,35,5
```

```
            I=0
            J=0
            NN=N+4
            LM=N+40
            DO 5 I=1,N
                 X(I)=0.0
                 Y(I)=0.0
                 GAP(I)=0.0
                 G(I)=0.0
 5          CONTINUE
            DO 71 II=1,10
            DO 71 I=0,5
                  REJ(II,I)=0
 71         CONTINUE
            DO 81 JJ=1,10
            DO 81 J=1,5
                  PWR(JJ,J)=0.0
 81         CONTINUE

*     LATEX FORMATTING

      WRITE(NN,970),N
970   FORMAT('\\begin{table}[h]'/'\\caption{Power Study: Sample size '
     +,I2,', shape 0.5}')
      WRITE(NN,982)
982   FORMAT('\\begin{center}')
      WRITE(NN,971)
971   FORMAT('\\begin{tabular}{||c|c|c|c|c|c||} \\hline')
      WRITE(NN,983)
983   FORMAT('Distribution&$\\alpha=0.20$&$\\alpha=0.15$&$\\alpha='
     +'0.10$&$\\alpha=0.05$&$\\alpha=0.01$\\zline',/,'\\hline')

      READ(N,*)(MUDIF(I),I=1,N-1)
C      WRITE(33,903)(MUDIF(I),I=1,N-1)
          DO 100 DIST=1,NUMDIST
                 DO 6 J=1,IREPS
                      ZSTAR(J)=0.0
  6              CONTINUE
              DO 110 J=1,IREPS
                        GO TO(200,210,220,230,240,250,
     +                        260,270,280,290),DIST
                   WRITE(*,101)
 101               FORMAT( 'ERROR AT COMPUTED GOTO')
```

```
      STOP
200   CALL RNGAM(N,K,Y)
      GO TO 440
210   CALL RNGAM(N,1.5,Y)
      GO TO 440
220   CALL RNGAM(N,2.5,Y)
      GO TO 440
230   CALL RNGAM(N,4.0,Y)
      GO TO 440
240   CALL RNWIB(N,2.0,Y)
      GO TO 440
250   CALL RNWIB(N,3.0,Y)
      GO TO 440
260   CALL RNLNL(N,0.0,2.0,Y)
      GO TO 440
270   CALL RNLNL(N,1.0,1.0,Y)
      GO TO 440
280   CALL RNBET(N,2.0,2.0,Y)
      GO TO 440
290   CALL RNBET(N,2.0,3.0,Y)
440   DO 7 I=1,N
      X(I)=Y(I)+10.0
7     CONTINUE
      CALL SVRGN(N,X,XX)

*     THIS ROUTINE IS ALSO USED FOR CENSORED SAMPLES
*     CENSOR IS SET TO 1.0 FOR COMPLETE SAMPLES

      DO 8 I=1,N*CENSOR-1
      GAP(I)=XX(I+1)-XX(I)
8     CONTINUE
      DO 9 I=1,N*CENSOR-1
      G(I)=GAP(I)/MUDIF(I)
9     CONTINUE
      NUMSUM=0.0
      DO 10 I=1,N*CENSOR-2
      NUMSUM=NUMSUM+(N-1-I)*G(I)
10    CONTINUE
      NUM=2.0*NUMSUM
      DENSUM=0.0
      DO 11 I=1,N*CENSOR-1
      DENSUM=DENSUM+G(I)
11    CONTINUE
```

```
                  DENOM=(N-2)*DENSUM
                  ZSTAR(J)=NUM/DENOM
 110           CONTINUE
          CALL SVRGN(IREPS,ZSTAR,ZSTAR)
          DO 120 I=1,IREPS
               IF(ZSTAR(I).GT.CRIT(N)) THEN
                  DO 99 J=1,5
                  REJ(DIST,J)=REJ(DIST,J)+1
  99              CONTINUE
                ELSEIF(ZSTAR(I).GT.CRIT(N-1)) THEN
                  DO 95 J=1,4
                  REJ(DIST,J)=REJ(DIST,J)+1
  95              CONTINUE
                ELSEIF(ZSTAR(I).GT.CRIT(N-2)) THEN
                  DO 90 J=1,3
                  REJ(DIST,J)=REJ(DIST,J)+1
  90              CONTINUE
                ELSEIF(ZSTAR(I).GT.CRIT(N-3)) THEN
                  DO 85 J=1,2
                  REJ(DIST,J)=REJ(DIST,J)+1
  85              CONTINUE
                ELSEIF(ZSTAR(I).GT.CRIT(N-4)) THEN
                  REJ(DIST,1)=REJ(DIST,1)+1
                ELSE
                  REJ(DIST,0)=REJ(DIST,0)+1
               ENDIF
 120      CONTINUE
               DO 130 J=1,5
               PWR(DIST,J)=(REAL(REJ(DIST,J))/
    +          REAL(IREPS))
 130           CONTINUE
          WRITE(NN,972)D(DIST),(PWR(DIST,J),J=1,5)
 972      FORMAT(A14,5(' & ',F6.3),'\\','\\\\hline')
 100      CONTINUE
        CLOSE(N)
        WRITE(NN,973)
 973    FORMAT('\\end{tabular}'/'\\end{center}'/'\\end{table}')
        CLOSE(NN)
2000   CONTINUE
       STOP
1000    WRITE(*,1001) IERROR
1001    FORMAT(1X,'IOSTAT = ', I3/)
      END
```

Appendix G. Regression Analysis for Complete and Censored Samples

This regression function represents the complete or censored samples with shape parameters between 0.5 and 4.0. It is valid for sample sizes between 5 and 35, and for significance levels between 0.20 and 0.01.

1.50.3

```
STEPWISE REGRESSION OF CRITICAL VALUES

A : Censor %                 AB :Censor % * Sample size
B : Sample Size              AD :Censor % * Significance level
C : Shape Parameter          BLOG : LOG(Sample size)
D : Significance level       CLOG : LOG(Shape Parameter)

                                                  B C D
                                                  L L L
                                              A A B O O O
STEP    R SQ        MSE         T           A B D D G G G
----   ------   ---------    -------      -------------
  1    0.0000     0.02784                 . . . . . . .
  2    0.4440     0.01551   -21.11 +      . B . . . . .
  3    0.7022     0.00832   -21.98 +      . B C . . . .
  4    0.8265     0.00486   -19.96 +      A B C . . . .
  5    0.9242     0.00213   -26.75 +      A B C . E . .
  6    0.9521     0.00135    17.95 +      A B C D E . .
  7    0.9715   8.008E-04   -19.45 +      A B C D E . G
  8    0.9803   5.545E-04   -15.71 +      A B C D E F G
RESULTING STEPWISE MODEL
VARIABLE    COEFFICIENT    STD ERROR     STUDENT'S T      P      VIF
---------   -----------    ---------     -----------    -----    ---
CONSTANT       2.77599       0.01899        146.17       0.00
A             -0.00905     1.436E-04        -63.06       0.00     2.1
AB           4.671E-05     4.024E-06         11.61       0.00    14.1
AD            -0.01238     5.008E-04        -24.71       0.00     9.9
BD             0.04137       0.00142         29.21       0.00     6.8
BLOG          -0.67283       0.01246        -53.99       0.00    11.9
CLOG          -0.05469       0.00348        -15.71       0.00     1.0
DLOG          -0.12667       0.00542        -23.37       0.00     6.4
CASES INCLUDED  560       R SQUARED  0.9803          MSE 5.545E-04
MISSING CASES     0       ADJ R SQ   0.9801          SD     0.02355
```

SOURCE	DF	SS	MS	F	P
REGRESSION	7	15.2576	2.17965	3930.97	0.0000
RESIDUAL	552	0.30607	5.545E-04		
TOTAL	559	15.5636			

STEPWISE ANALYSIS OF VARIANCE OF E

SOURCE	INDIVIDUAL SS	CUM DF	CUMULATIVE SS	CUMULATIVE MS	ADJUSTED R-SQUARED	MALLOWS' CP
CONSTANT	979.170					
A	4.92988	1	4.92988	4.92988	0.3155	18621.8
AB	4.84713	2	9.77701	4.88851	0.6269	9882.1
AD	3.08646	3	12.8635	4.28782	0.8256	4317.7
BD	0.34264	4	13.2061	3.30153	0.8474	3701.8
BLOG	1.61175	5	14.8179	2.96357	0.9516	797.0
CLOG	0.13679	6	14.9546	2.49244	0.9604	552.3
DLOG	0.30293	7	15.2576	2.17965	0.9801	8.0
RESIDUAL	0.30607	559	15.5636	0.02784		

Appendix H. Regression Analysis for Shape Parameter $\alpha = 0.5$, and Complete Samples

B : Sample size
D : Significance level
BLOG : LOG(Sample size)
DSIN : SIN(Significance level)
DLOG : LOG(Significance level)

STEP	R SQ	MSE	T	BD	BLOG	DLOG	DSIN
1	0.0000	0.02265		.	.	.	.
2	0.5954	0.00944	-6.97 +	A	.	.	.
3	0.6734	0.00786	-2.76 +	A	B	.	.
4	0.9472	0.00131	-12.68 +	A	B	.	D
5	0.9845	3.982E-04	-8.49 +	A	B	C	D

RESULTING STEPWISE MODEL

VARIABLE	COEFFICIENT	STD ERROR	STUDENT'S T	P	VIF
CONSTANT	1.76451	0.04327	40.78	0.0000	
BD	0.04483	0.00460	9.74	0.0000	6.2
BLOG	-0.50987	0.02047	-24.90	0.0000	2.8
DLOG	-0.16220	0.01910	-8.49	0.0000	6.9
DSIN	-1.38730	0.16100	-8.62	0.0000	10.4

SOURCE	DF	SS	MS	F	P
REGRESSION	4	0.75819	0.18955	476.02	0.0000
RESIDUAL	30	0.01195	3.982E-04		
TOTAL	34	0.77014			

STEPWISE ANALYSIS OF VARIANCE OF E

SOURCE	INDIVIDUAL SS	CUM DF	CUMULATIVE SS	CUMULATIVE MS	ADJUSTED R-SQUARED	MALLOWS' CP
CONSTANT	57.1557					
BD	0.45855	1	0.45855	0.45855	0.5832	751.5
BLOG	0.06002	2	0.51857	0.25929	0.6529	602.8
DLOG	0.21005	3	0.72863	0.24288	0.9409	77.2
DSIN	0.02956	4	0.75819	0.18955	0.9824	5.0
RESIDUAL	0.01195	34	0.77014	0.02265		

R-SQUARED	0.9845	RESID. MEAN SQUARE (MSE)	3.982E-04
ADJUSTED R-SQUARED	0.9824	STANDARD DEVIATION	0.01995

1.51.2

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13. ABSTRACT *(Maximum 200 words)*

This thesis studies a new goodness-of-fit test for the gamma distribution with known shape parameter. This test statistic, Z^*, is based on spacings from complete or censored samples. The size of samples varied between 5 and 35. The critical value tables were generated for the Z^* test statistic for complete and censored samples. The critical values were obtained for five different significance levels: 0.20, 0.15, 0.10, 0.05, and 0.01. An extensive power study, containing 50,000 Monte Carlo runs, was conducted using nine alternative distributions, H_a. It was observed that the Z^* test statistic was more powerful against certain alternatives which are less skewed than the gamma distribution with a given shape parameter. A regression between the critical values and the sample size, shape parameter, significance levels and degree of censoring was established. The power of the Z^* test statistic is compared to the powers of the competing test statistics (K-S, W^2, and A-D). This thesis reveals that the Z^* test statistic is a directional test. This feature may be utilized to attain higher power values by coupling the Z^* and the A-D test statistics in a sequential test.

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